Reaction dynamics around the Coulomb barrier with weakly bound Radioactive Ion Beams: elastic scattering cross sections for the systems $^{17}\text{F} + ^{208}\text{Pb}$ and $^{11}\text{Be} + ^{209}\text{Bi}$

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L'unico fine della scienza è l'onore dello spirito umano.
Jacobi, Lettera a Legendre 3 luglio 1830
The only purpose of the science is the honour of the human spirit.
Jacobi, Letter to Legendre July, 3rd 1830

A tutti coloro che mi sono,
mi sono stati e mi saranno vicini.
To all the people who are,
were and will be close to me.
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Introduction

Over the last decades, the study of nuclear reactions induced by light weakly bound nuclei has attracted the interest of many scientific communities around the world. Recent developments together with a larger availability of radioactive ion beams (RIB’s) should give the possibility to observe new phenomena. In fact exotic nuclei might have extremely different properties from those reported for nuclei along the valley of stability and, therefore, they are expected to behave differently from stable well-bound nuclei, especially at Coulomb barrier energies.

In this energy range, the most relevant process, besides the scattering, is the fusion of the two interacting nuclei. In case of collisions between stable nuclei, this process could be strongly influenced by projectile or target inelastic excitations and by transfer reactions. In reactions involving loosely bound nuclei new degrees of freedom come into the game, namely:

- the **halo** structure. The last loosely bound nucleons (mostly in s-states) have wave functions extended well outside the nuclear core. This implies an r.m.s. radius larger than what obtained from the systematics of stable nuclei. Such structures have been identified in several light nuclei as, for example, $^{11}$Be [Tan85] and $^{11}$Li [Tan96].

- the **neutron skin** structure. The last neutrons are mostly orbiting in the outer part of the nucleus in a skin-like structure. This is the case of the $^6$He [Tan92] and $^8$He nuclei [Alk97].

- the **small binding energy** of the last nucleons. This is a very general feature especially for RIB’s at the drip-lines. Typical binding energy are about one order of magnitude smaller than for nuclei close to the stability valley.

All these phenomena are not independent from each other and are expected to generate opposite and sometimes conflicting effects. For example, the halo structure leads to a radius larger than the systematics, which means a reduced Coulomb barrier height and a consequent enhancement of the fusion cross
section. On the other side, a small binding energy could remove flux from the fusion channel, since the projectile could easily break before the fusion process is complete, thus increasing the reaction cross section. A critical overview of recent experiments performed to study the fusion reactions at Coulomb barrier energies with weakly bound RIB’s is presented in [Sig01b].

Our interest for the physics exploited by RIB’s have motivated the study of the scattering process for the systems $^{17}$F + $^{208}$Pb and $^{11}$Be + $^{209}$Bi in the energy range around the Coulomb barrier. Both projectiles are rather interesting. In fact the $^{17}$F ($T_{1/2} = 64.5$ s) nucleus (i) is the lightest particle-stable Fluorine isotope with a proton binding energy of 601 keV, (ii) has only one bound state below the breakup threshold, and (iii) the $l = 0$ structure of its first excited state makes this level a “good” proton halo candidate [Bor93]. Experimental measurements report for this state an r.m.s. radius of 5.33 fm [Mor97] compared to the 3.7 fm for the ground state. Also the $^{11}$Be ($T_{1/2} = 13.8$ s) ion shows some peculiar features: (i) it has a well-known neutron halo structure [Tan85], (ii) its binding energy is 504 keV and (iii) it has only one excited level below the particle threshold.

In order to perform an experiment with radioactive projectiles, a primary nuclear reaction is necessary to produce them and a high efficiency beam transport system is needed to separate the selected ions from spurious reaction products and to focus them on a secondary target. Up to now RIB’s delivered from existing facilities suffer of low beam intensity and poor energy resolution and emittance. Therefore measurements performed with present RIB’s have limited statistics and it is usually very hard to evidence at the same time new phenomena and well-known processes with statistical significance. For these reasons, the study of nuclear reactions induced by light stable weakly bound projectiles, such as for example $^6$Li and $^9$Be (the most weakly bound nuclei among all the stable ones), could be very useful since these nuclei might behave similarly to light RIB’s. Chapter 1 will present a brief overview of the most recent experiments involving light loosely bound projectiles in the energy range around the Coulomb barrier.

The study of the RIB’s exotic features using such low intensity beams requires the design of detector arrays with large solid angle coverage and energy and position resolution good enough to guarantee the complete kinematic reconstruction of the events. These guidelines have brought us to design and develop a new detector array named EXODET (EXOtic DETector), described in Chapter 2. This apparatus uses a very innovative readout electronics based on the ASIC chipset [Par99]. Such kind of device, originally developed for high-energy particle physics experiments, was found to be suitable for our purposes and for the first time was adapted to low-energy nuclear physics measurements.
Chapter 3 will present the $^{17}$F secondary beam production at ANL (USA) via the In-Flight technique and the beam optics used to separate the contaminant $^{17}$O primary beam and to focus the $^{17}$F ions on the $^{208}$Pb target. We will then describe the experimental set-up and the procedure adopted in the data analysis both for the scattering and the exclusive breakup $^{17}$F $\rightarrow$ $^{16}$O $+ p$ cross sections.

The experiment performed at RIKEN (Japan) to study the $^{11}$Be scattering process from a $^{209}$Bi target will be discussed in Chapter 4. We will first describe the secondary beam production via fragmentation of a high-energy $^{13}$C primary beam and the beam optics of the RIPS spectrometer [Kub92] used to select the required isotopes among all the projectile fragments. We will then present all the equipments necessary for slowing down and tagging the $^{11}$Be secondary beam, the experimental set-up and all the details for the evaluation of the scattering angular distributions.

The theoretical analysis will be described in Chapter 5. The experimental data have been analyzed within the optical model framework in order to get consistent potential parameters sets, to determine the real and the imaginary strong absorption radius (only for the system $^{17}$F $+ ^{208}$Pb), to calculate the reaction cross sections and to compare these results with those obtained for similar mass systems in the energy range around the Coulomb barrier.

The discussion of the results will be undertaken in Chapter 6. The reaction cross sections deduced from the elastic scattering will be compared with those measured for the other open channels in this energy range, namely fusion, inclusive and exclusive breakup processes. In particular we will check whether the sum of these cross sections exhausts the reaction cross section. Some hypotheses on the processes responsible for the possible discrepancies will be finally given.

Some conclusive comments will be drawn in Chapter 7.
Introduzione

Negli ultimi anni lo studio delle reazioni nucleari indotte da nuclei leggeri debolmente legati ha attratto l’interesse di numerose comunità scientifiche. I recenti sviluppi e la maggiore disponibilità di Fasci Radioattivi Accelerati (RIB's, dall’inglese Radioactive Ion Beams) dovrebbero fornire la possibilità di osservare nuovi fenomeni. Infatti i nuclei esotici potrebbero presentare delle caratteristiche e dei comportamenti completamente diversi da quelli osservati per nuclei prossimi alla valle della stabilità, specialmente nel caso di reazioni ad energie attorno alla barriera coulombiana.

In questo intervallo energetico il processo dominante, oltre ovviamente alla diffusione elastica, è la fusione dei due nuclei interagenti. Nel caso di collisioni tra nuclei stabili, questo processo può essere fortemente influenzato dalle eccitazioni inelastiche, sia del proiettile che del bersaglio e dal trasferimento di nucleoni. Invece, nelle reazioni nucleari che coinvolgono nuclei debolmente legati entrano in gioco nuovi gradi di libertà, ovvero:

- l’alone nucleare. I nucleoni più esterni, principalmente orbitanti in stati di onda s, sono debolmente legati e hanno delle funzioni d’onda estese ben oltre il core nucleare. Il raggio nucleare risulta pertanto maggiore di quello previsto dalla sistematica dei nuclei stabili. Strutture di questo tipo sono state riscontrate in alcuni nuclei leggeri, per esempio $^{11}$Be [Tan85] e $^{11}$Li [Tan96].

- la neutron skin structure. Gli ultimi neutroni orbitano principalmente nelle zone più esterne del nucleo originando un maggiore addensamento superficiale di neutroni, ovvero una neutron skin structure. Rientrano in questa tipologia di nuclei, per esempio, $^{16}$He [Tan92] e $^{18}$He [Alk97].

- la debole energia di legame degli ultimi nucleoni. Si tratta di una caratteristica comune a molti RIB’s prossimi alle drip-line, per i quali l’energia di legame è all’incirca un ordine di grandezza inferiore a quella dei nuclei prossimi alla valle della stabilità.
Tutti questi fenomeni sono molto spesso interconnessi e talvolta generano effetti opposti. Per esempio l’alone nucleare implica un raggio maggiore di quello stimato dalla sistematica, con una conseguente diminuzione della barriera coulombiana ed un aumento della sezione d’urto di fusione. D’altro lato, una debole energia di legame potrebbe sfavoreire questo canale di reazione, dal momento che il proiettile potrebbe rompersi prima che il processo di fusione possa essere completato. Una panoramica critica degli ultimi esperimenti sul processo di fusione indotto da nuclei radioattivi debolmente legati è raccolta in [Sig01b].

Il nostro interesse per questo tipo di ricerca ha motivato lo studio dei processi di diffusione elastica nei sistemi $^{17}$F + $^{208}$Pb e $^{11}$Be + $^{209}$Bi nell’intervallo energetico attorno alla barriera coulombiana. Entrambi i proiettili sono particolarmente interessanti, infatti il $^{17}$F ($T_{1/2} = 64.5$ s) (i) è l’isotopo più leggero del Fluoro stabile per emissione di particelle (l’energia di separazione del proton $S_p$ è pari a $601$ keV), (ii) ha un solo stato legato al di sotto della soglia di breakup e (iii) la struttura $l = 0$ del suo primo livello eccitato lo rende un buon candidato a costituire uno nucleo con alone protonico [Bor93]. Le misure sperimentali riportano per questo stato un raggio quadratico medio di 5.33 fm [Mor97] superiore di molto al valore misurato per lo stato fondamentale (3.7 fm). Anche il $^{11}$Be ($T_{1/2} = 13.8$ s) ha delle caratteristiche peculiari dal momento che (i) presenta un’evidente alone neutrontico [Tan85], (ii) ha una bassa energia di legame ($S_n = 504$ keV) ed (iii) ha un solo livello eccitato al di sotto della soglia di breakup.

Entrambe le reazioni che abbiamo studiato sono indotte da proiettili radioattivi. In questo tipo di esperimenti, abbiamo innanzitutto bisogno di una reazione nucleare primaria che possa i nuclei richiesti e successivamente di un efficiente sistema di trasporto che permetta la soppressione dei prodotti spuri di reazione e la focalizzazione del fascio secondario. I fasci radioattivi prodotti fino ad oggi dalle facility esistenti sono poco intensi ed hanno una scarsa risoluzione sia energetica che posizionale. Pertanto gli esperimenti che coinvolgono RIB’s hanno bassa statistica ed è spesso molto difficile evidenziare al tempo stesso nuovi fenomeni e processi ben noti con una sufficiente accuratezza. Per questo motivo, lo studio delle reazioni nucleari indotte da proiettili leggeri, stabili e debolmente legati, ad esempio $^6$Li e $^9$Be (i due nuclei stabili più debolmente legati che esistono in natura), potrebbe risultare molto utile dal momento che questi nuclei potrebbero presentare comportamenti simili a quelli dei fasci radioattivi leggeri. Il Capitolo 1 presenterà una breve panoramica dei più recenti risultati sperimentali ottenuti con questo tipo di proiettili nell’intervallo energetico attorno alla barriera coulombiana.

Lo studio delle caratteristiche dei nuclei esotici utilizzando fasci di bassa intensità richiede il design di array di rivelatori che sottendano un elevato
angolo solido e che abbiano una risoluzione energetica e posizionale tale da garantire una ricostruzione cinematica completa degli eventi rivelati. Queste linee guida ci hanno condotto al design e allo sviluppo di un nuovo apparato sperimentale denominato EXODET (EXOtic DETector), descritto nel Capitolo 2. EXODET utilizza un’elettronica di lettura molto innovativa basata sul chip ASIC [Par99]. Questo tipo di chip è stato originariamente sviluppato per esperimenti di fisica particellare ed è stato adattato per la prima volta ad un esperimento di fisica nucleare delle basse energie.

Il Capitolo 3 presenterà la tecnica di produzione del fascio secondario di $^{17}$F ad ANL (USA) tramite la tecnica In-Flight e l’ottica utilizzata per separare il contaminante fascio primario di $^{17}$O e per focalizzare il $^{17}$F sul bersaglio di $^{208}$Pb. Descriveremo inoltre il set-up sperimentale e la procedura di analisi adottata per determinare la sezione d’urto dei processi di scattering elastico e di breakup esclusivo $^{17}$F → $^{16}$O + $p$.

L’esperimento svolto presso i laboratori di RIKEN (Giappone) per studiare il processo di diffusione elastica di nuclei di $^{11}$Be da un bersaglio di $^{209}$Bi verrà discusso nel Capitolo 4. Descriveremo dapprima la tecnica di produzione del fascio secondario tramite frammentazione di un fascio primario molto energetico di $^{13}$C e l’ottica dello spettrometro RIPS [Kub92] utilizzato per selezionare gli isotopi desiderati tra i vari frammenti del proiettile. Presenteremo successivamente la strumentazione utilizzata per frenare ed etichettare il fascio secondario, il set-up sperimentale e tutti i dettagli della misura delle distribuzioni angolari dello scattering elastico.

L’analisi teorica verrà descritta nel Capitolo 5. I dati sperimentali sono stati analizzati al fine di calcolare dei consistenti parametri di modello ottico, di determinare i raggi di massimo assorbimento reali ed immaginari (solo per il sistema $^{17}$F + $^{208}$Pb), di calcolare le sezioni d’urto di reazione e di confrontare i risultati con quelli ottenuti per sistemi simili ad energie attorno alla barriera coulombiana.

La discussione dei risultati ottenuti verrà intrapresa nel Capitolo 6. Le sezioni d’urto di reazione dedotte dalle distribuzioni angolari degli eventi di scattering elastico verranno confrontate con quelle ottenute per gli altri canali di reazione in questo stesso intervallo di energia, cioè la fusione ed i processi di breakup inclusivo ed esclusivo. In particolare si verificerà se la somma delle sezioni d’urto misurate esaurisce o meno la sezione d’urto di reazione e verranno fatte alcune ipotesi sui processi responsabili di eventuali discrepanze.

Alcuni commenti conclusivi verranno delineati nel Capitolo 7.
Chapter 1

The physics case

The availability of Radioactive Ion Beams opened a completely new field of research in Nuclear Physics. In particular for the interaction dynamics at the Coulomb barrier of light RIB’s there are some new features, absent with nearly all stable beams, that have to be taken into account. Many light RIB’s are much more loosely bound than stable nuclei. Typical binding energies range from 0.1 to 0.5 MeV, compared to the 6-8 MeV average nucleon separation energy for heavier nuclei close to the valley of stability. In addition, in many cases, the weakly bound valence nucleons give rise to a halo structure. It means that the r.m.s. radius is larger than what established for most of the stable nuclei by the well-known formula $R = r_0 A^{1/3}$, with $r_0 \sim 1.2$ fm. Fig. 1.1 shows that the $^{11}\text{Li}$ halo nucleus has a nuclear radius as large as the $^{48}\text{Ca}$ nuclear radius. Halo structure and small binding energy are closely related to each other since the first effect is often a direct consequence of the second one.

The influence of these two features on the reaction dynamics at Coulomb barrier energies could give quite different results. In fact the presence of a nuclear halo implies a lowering of the Coulomb barrier and therefore the fusion probability, exponentially depending on the height of the barrier, is expected to be larger. On the other side, the small binding energy enhances the projectile breakup probability and consequently the fusion cross section should be reduced or alternatively the coupling to possible strong breakup channels in the continuum could increase the fusion through such doorway states.

We clearly see that there is a large variety of opportunities, which has triggered quite many theoretical and experimental works. Theory has sometimes produced conflicting predictions, while up to now experiments performed with light RIB’s, such as $^{6}\text{He}$, $^{11}\text{Be}$, $^{17}\text{F}$, suffer of low statistics. In fact the highest RIB’s intensities presently achieved are $\sim 3$ orders of mag-
Figure 1.1: Effects of the halo structure on the nuclear radius: comparison between the $^{208}$Pb and $^{48}$Ca nuclear radii and that of the halo nucleus $^{11}$Li.

magnitude smaller than with stable beams. This fact does not help to clearly distinguish between different theoretical predictions and, in particular, to establish whether the fusion cross section at the Coulomb barrier is enhanced or hindered.

The experiments performed with $^6$He beam on $^{209}$Bi [Agu00], $^{64}$Zn [DiP04] and $^{63}$Cu [Nav04] showed a fairly large $\alpha$-particle production in the energy range around the Coulomb barrier. A straightforward interpretation is that $\alpha$-particles originate from the breakup channel $^6$He $\rightarrow$ $\alpha + n + n$ ($S_{2n} = 0.901$ MeV). Data collected with higher statistical accuracy for the two most weakly bound stable nuclei $^8$Be ($S_n = 1.573$ MeV) [Sig01a] and $^6$Li ($S_d = 1.475$ MeV) [Sig01c] interacting with high-Z targets also showed the presence of very strong “inclusive” $\alpha$ production channels. With the term “inclusive” we mean that the $\alpha$-particles were detected regardless the coincidences with other fragment(s) of the incoming particles.
Figure 1.2: Cross sections for the “inclusive” α-particle production (“inclusive breakup”) compared with the “complete” fusion cross section for the systems $^6\text{Li} + ^{208}\text{Pb}$, $^9\text{Be} + ^{209}\text{Bi}$ and $^6\text{He} + ^{209}\text{Bi}$.

Fig. 1.2 shows the comparison between the “inclusive” α-particle production cross section and the “complete” fusion cross section, i.e. the fusion of all the projectile nucleons with the target, for the systems $^6\text{Li} + ^{208}\text{Pb}$ [Sig01c], $^9\text{Be} + ^{209}\text{Bi}$ [Sig01a] and $^6\text{He} + ^{209}\text{Bi}$ [Agu00]. In all cases there is a strong reaction channel, comparable to and, at times, larger than the fusion channel. This process has been named “inclusive” breakup. A lighter system like $^6\text{Li} + ^{28}\text{Si}$ [Pak03] presents a very similar scenario.

This reaction mechanism has never been observed with well-bound projectiles and for this reason it should be closely related to the expected large breakup probability for these loosely bound nuclei. From Fig. 1.3 we can also observe that, for the three systems, the sum of “inclusive breakup” and “complete” fusion channels exhausts the total reaction cross section deduced from elastic scattering data in the energy range around the Coulomb barrier.

Whether and how the breakup probability influences the whole reaction dynamics and, in particular, the fusion process is still on open question. To enlight the situation, let us now briefly review the results obtained for the systems $^9\text{Be} + ^{208}\text{Pb}$, $^{209}\text{Bi}$ and $^6\text{Li} + ^{208}\text{Pb}$.
1.1 The systems $^9\text{Be} + ^{208}\text{Pb}, ^{209}\text{Bi}$

The fusion cross section for the systems $^9\text{Be} + ^{208}\text{Pb}, ^{209}\text{Bi}$ [Hin99, Sig99, Yos96] resulted to be $\sim 30\%$ smaller than the theoretical estimates obtained from the ultimate fusion codes at Coulomb barrier energies. This fusion suppression has been extensively investigated and its origin was attributed to the breakup process $^9\text{Be} \rightarrow \alpha + \alpha + n$. In fact, as we have already observed, a very large yield of $\alpha$-particles, i.e. projectile fragments, have been measured for both systems in this energy range. These reaction products might also interact with the target, originating a new process called “incomplete” fusion, i.e. a process where only some nucleons of the projectile are captured by the target. In fact for the system $^9\text{Be} + ^{208}\text{Pb}$ the incomplete fusion of He-fragments with the target accounts just for the 30% reduction of the complete fusion probability. On the other side, Fig. 1.4 reports a fairly large production of $^{212}\text{At}$ nuclei, mainly arising from the incomplete fusion of $\alpha$-particles, for the system $^9\text{Be} + ^{208}\text{Pb}$.

The main conclusion is that for these two systems the “complete” fusion (CF) is reduced if compared with that expected for well bound systems, where the breakup effects can be excluded. However, at least for the system $^9\text{Be} + ^{208}\text{Pb}$, if we take also into account the “incomplete” fusion (IF) cross section, the “total” fusion process (CF + IF) comes back to that of a well
bound system.

At present, the origin of the incomplete fusion process for nuclear reactions involving loosely bound projectiles still remains an open question. In a very naive description we can identify two possible reaction mechanisms:

1. a classical direct process (like in a \((d,p)\) reaction),

2. absorption of one or more projectile nucleons after the breakup process has occurred at large (small) impact parameters due to Coulomb (nuclear) interaction.

This second process has been known under several names: stripping breakup, partial fusion, diffractive dissociation, ...

1.2 The system \(^6\text{Li} + ^{208}\text{Pb}\)

Several sets of data with high statistical accuracy have been measured for this system: elastic scattering [Kee94], complete fusion [Wu03], inclusive breakup [Sig01c] and exclusive breakup [Sig03]. Fig. 1.5 gives a synoptic view of all the data. In this figure with “inclusive breakup-\(\alpha (-d)\)” we mean the cross section for \(\alpha (d)\) production regardless of any other simultaneous process,
while the curve “exclusive breakup” indicates the cross section for the $\alpha + d$ coincidence events.

![Graphical representation of cross sections](image)

**Figure 1.5:** Experimental cross sections for various breakup processes, complete fusion and total reaction cross section deduced from elastic scattering measurements.

We can observe that:

1. “inclusive breakup-$\alpha$” cross section is larger than complete fusion one and their sum adds up to the total reaction cross section, as already seen in Fig. 1.3;

2. exclusive breakup process has a cross section one order of magnitude smaller than the “inclusive breakup-$\alpha$”;

3. “inclusive breakup-$d$” has a cross section one order of magnitude smaller than the “inclusive-$\alpha$”.

From all these observations we may just conclude that in the disintegration (breakup) of $^6$Li nuclei in $\alpha + d$, it is more probable to have only one fragment in the exit channel rather than both. In addition, the probability to be captured from the target is higher for the lighter fragment (the deuteron in our case).
1.3 Why the systems $^{17}\text{F} + ^{208}\text{Pb}$ and $^{11}\text{Be} + ^{209}\text{Bi}$?

Within this scenario we have undertaken the study of the elastic scattering differential cross sections for the systems $^{17}\text{F} + ^{208}\text{Pb}$ and $^{11}\text{Be} + ^{209}\text{Bi}$ at Coulomb barrier energies. These experiments constitute basic measurements to determine the interaction potential between the colliding nuclei and to investigate the influence of the projectile halo structure and low binding energy on the reaction dynamics. In addition, another important piece of information that can be extracted from elastic scattering data is the total reaction cross section. This is a very useful tool, in fact we have already observed that for similar systems ($^{9}\text{Be} + ^{209}\text{Bi}, ^{6}\text{Li} + ^{208}\text{Pb}$ and $^{6}\text{He} + ^{209}\text{Bi}$) the sum of fusion and inclusive breakup cross section essentially exhausts the reaction cross section in this energy range. Therefore, since the fusion processes have already been measured for both systems [Reh98, Sig04], our experiments could give a clear indication on the contribution of a possible inclusive breakup channel to the total reaction cross section.

In addition, the results obtained for these systems can be compared with those obtained for similar mass systems in the same energy range. In particular, the $^{17}\text{F}$ behavior can be compared with that of the well-bound nuclei $^{16}\text{O}$, $^{17}\text{O}$ and $^{19}\text{F}$, where no breakup effects are expected, in order to investigate the influence of the low binding energy of the last proton on the whole interaction. Moreover, since $^{17}\text{F}$ breaks in well-bound charged fragments $^{16}\text{O}$ and $p$, the collected data should also give the opportunity to provide a first direct measurement of the $^{17}\text{F}$ exclusive breakup cross section at energies around the Coulomb barrier.

On the other hand, the $^{11}\text{Be}$ nucleus can be directly compared either with the stable $^{9}\text{Be}$ isotope, that is very loosely bound too, or with other weakly bound projectiles, such as $^{6}\text{Li}$, $^{8}\text{Li}$ and $^{6}\text{He}$. In this case it would be very interesting to see if, even for this system, a very large “inclusive breakup” cross section is predicted and the halo structure influence on the reaction dynamics.
Chapter 2

Experimental apparatus: EXODET

Up to now the Radioactive Ion Beams delivered by the first generation of RIB’s facilities are characterized by very low beam intensity \((10^5-10^6 \text{ pps})\) and poor energy resolution and emittance. For these reasons, the study of the RIB’s exotic features requires detector systems subtending the largest possible solid angle. Moreover, a high energy resolution and position sensitivity are also necessary to provide a complete event kinematics reconstruction of the reaction.

Figure 2.1: Schematic arrangement of the 8 EXODET telescopes around the target. In yellow the inner \(\Delta E\) detectors, in orange the outer \(E\) detectors.
2.1 Arrangement

To fulfill these requirements, a new apparatus for charged particle detection and identification named EXODET (an acronym for EXOtic DETector) has been designed and developed. The basic EXODET module is a large active area silicon detector, produced by MICRON Semiconductors Ltd. Each detector has an active area of $(50 \times 50)$ mm$^2$ and a total occupancy of $(53 \times 53)$ mm$^2$, considering the guard rings and the surrounding passivated region. Each detector front side is segmented in 100 strips with a pitch size of 0.5 mm and a separation distance of 50 $\mu$m. Each strip is wire-bonded to an external gold-plated copper pin. The rear side of the detector is a unique electrode.

![Diagram of EXODET apparatus]

Figure 2.2: Schematic drawing of the EXODET forward cube assembly. The strips of the $\Delta E$ layer are perpendicular to the beam direction, while the strips of the $E$ layer have the same direction of the beam. The detector support boards (drawn in green) contain the position readout chip and the related electronics.

The whole EXODET array consists of 16 detectors, arranged in 8 two-stage telescopes, allowing Z identification of the particles passing through the first layer, by means of the standard $\Delta E - E$ technique. Detectors with different thicknesses can be used. In the current configuration the inner detector is 70(40) $\mu$m thick and the outer is 500 $\mu$m. Fig. 2.1 shows the displacement of the telescopes around the target in the forward and backward hemispheres with respect to the incoming beam and the target position. In the present configuration the telescopes are arranged in two cubes with $\sim 5$ cm edge. Fig. 2.2 illustrates that the strips of the $\Delta E$ ($E$) detectors are orthogonal (parallel) to the beam direction. In such a way, it is possible to reconstruct the position of the particles passing through the $\Delta E$ detector.
with a pixel accuracy of $\sim (0.5 \times 0.5)$ mm$^2$. Hydrogen isotopes with incident energies higher than 2.3-4.2 MeV can be fully identified (energy, position, mass and charge) and this is still possible for heavier ions but their energy has to be at least 8 MeV, as shown in Table 2.1.

<table>
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<tr>
<td>proton</td>
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<td>3.2 MeV</td>
</tr>
<tr>
<td>deuteron</td>
<td>3.0 MeV</td>
<td>4.2 MeV</td>
</tr>
<tr>
<td>$^4$He</td>
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<td>11.3 MeV</td>
</tr>
<tr>
<td>$^4$He</td>
<td>9.0 MeV</td>
<td>12.7 MeV</td>
</tr>
<tr>
<td>$^{11}$Be</td>
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<td>$^{16}$O</td>
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<td>104.2 MeV</td>
</tr>
<tr>
<td>$^{17}$F</td>
<td>80.2 MeV</td>
<td>119.2 MeV</td>
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Table 2.1: Maximum $\Delta E$ energy released by ions passing through an EXODET $\Delta E$ detector 40 μm and 70 μm thick. To consider the non-spherical shape of the array, the values were calculated for particles hitting the detector surface with an angle of 45°.

The detection pixels subtend different solid and scattering angles, depending on their position and distance from the beam position hit on the target. As we will see in Secs. 2.3-2.4, a Monte-Carlo code has been performed to evaluate the geometrical efficiency of the EXODET module. The total solid angle covered by the whole array estimated with the simulation code is $\sim 72\%$ of $4\pi$. The Monte-Carlo simulation is a very useful tool also for the cases when the particles are stopped in the first layer: in fact it is still possible to reconstruct the $\theta$ angular distribution, although with a higher indetermination due to the non-spherical shape of EXODET (see Sec. 2.3).

Fig. 2.3 shows a picture of two EXODET detectors before assembly to form a telescope. The detectors have already been glued to a board containing the detector-chip interface electronics and the preamplifier used for the treatment of the energy signals. The total occupancy of the whole array, as it possible to appreciate in Fig. 2.4, is very small: 350 mm in the beam direction and 230 mm in the other two (except cabling). This extreme compactness makes the system easy transportable from laboratory to laboratory and adaptable to more complex experimental set-ups. The array has been recently provided with a cooling system, using Peltier cells and water as cooling fluid, to ensure an appropriate control and stabilization of the detector temperature.
Figure 2.3: ΔE and E detectors photo together with the board containing an EXODET telescope chip and electronics before their assembly.

Figure 2.4: Picture of the whole EXODET array mounted on its own mechanical support.

2.2 Readout electronics

The EXODET readout system has to consider 16 channels for the processing of the energy signals from the backplane of the detectors, and 1600 channels (100 strips × 16 detectors) for the position information gathered from the segmented sides.
2.2.1 Energy signal treatment

Figure 2.5: Schematic of the electronic circuit for one of the EXODET detectors. Orange boxes represent homemade electronics or software, whereas yellow ones refer to commercial modules. DAQ system is an acronym for Data Acquisition system.

The information about the energy released by the impinging particles is obtained from the unsegmented rear side of each detector. The analog energy signal is carried out with a standard nuclear electronic chain, consisting of a home-made low-noise preamplifier named CHAPLIN (CHArge Preamplifier Low-noise INFN Napoli), a 16-channel programmable Spectroscopy Amplifier (SA CAEN model N568B) and a 32-channel Analog to Digital Converter (ADC SILENA model 9418/6-V). The amplified output is also sent to an Octal Constant Fraction Discriminator (CFD EG&G-ORTEC model CF8000) in order to obtain a logic signal, used for the trigger generation (see Fig. 2.5). The CHAPLIN preamplifier has been developed by the Electronic Service of the INFN - Sezione di Napoli and directly mounted on the detector support board.
2.2.2 Position information readout

Standard electronic solutions, such as resistive charge partition chains or delay lines, could not be used for the readout of the position information because of the large amount of strip signals to be analyzed and the consequent prohibitive costs. Therefore, a new readout system based on an Application Specific Integrated Circuit (ASIC) chipset [Par99] was used (see Fig. 2.6).

![ ASIC chip mounted on the supporting board and connected to the electronics. ]

Such a device, originally developed for high-energy particle physics experiments, has been found to be suitable also for our purposes, once it has been equipped with an appropriate detector-chip interface. In particular, a resistive attenuator, with an attenuator factor of 70, has been implemented to match the strip signal into the dynamical range of the chip. Moreover a pitch size adapter has been designed to reduce the pitch size from the 0.5 mm at the detector edge to the about 80 μm at the chip border. The chip and the related electronics, *i.e.* the communication lines for the chip I/O, for the energy signal output and for the power supplies, are located in a support board near the detector, to ensure the maximum noise reduction.

Each chip has a size of \((5.7 \times 8.3)\) mm\(^2\) (see Fig. 2.6) and drives up to 128 input channels. We use one chip for each detector, connecting the strips of the detector to the first 100 lines. Both the analogical and the digital treatment of the signals, coming from each strip, are contemporarily and separately performed inside the chip. Fig. 2.7 shows a scheme of the single channel electronics contained into the chip. The input signals are pre-amplified and shaped. Afterwards a sampling of the input line voltage is performed at a frequency of 15 MHz and compared to an externally settable threshold voltage \(V_{THRESH}\). When the input signal is higher than \(V_{THRESH}\), a bit of
2.2. Readout electronics

![Diagram of the ASIC chip](image)

**Figure 2.7:** Simplified electronic scheme of the ASIC chip used for the readout of the position information of the EXODET detectors.

The circular memory buffer is set to 1. If an external trigger command arrives to the chip, this sampling procedure is stopped and the digital circuitry analyzes the memory buffer to look for a bit set to 1. If found, the chip gives an output data stream containing:

1. the event identification header;
2. the identification number of the hit strips and, for each of them:
   - the Time over Threshold (ToT),
   - the Jitter Time (JT).

**Time over Threshold (ToT)**

ToT is the time, measured in units of clock cycles, spent by any input signal, after its amplification and shaping, over the threshold voltage $V_{\text{THRESH}}$. Since only 4 bits are available for the ToT counter, its spectrum is distributed only over 16 channels. Due to non-linear correspondence between the ToT and the signal amplitude, the ToT only gives a rough information on the energy lost by a particle in the strip. This information is very useful to disentangle an event when two particles with different energy ranges hit two strips of the same detector, as we will see in the analysis of the breakup process $^{17}\text{F} \rightarrow ^{16}\text{O} + p$ (see Sec. 3.6).

**Jitter Time (JT)**

The JT represents the time interval, measured in units of clock cycles, between the assertion of a valid trigger and the arrival of the particle input.
signal. JT is a sort of correlation time useful to distinguish among spurious events and the trigger correlated ones. In the present set-up, the chip conversion gain is 150 mV/νC, the signal memory buffer length is 12 μs and the signal analysis window is 2 μs. The best time resolution achievable with the chip used is 67 ns.

2.2.3 Front-End electronics

To readout our segmented detector array, several front-end electronic boards have been developed. Also a new data acquisition system, named VIPER (VME Interfaced to PCl EXODET), has been designed from scratch to manage the readout from homemade and commercial electronic modules. All these new developments have been based on the VME bus because its widespread use in nuclear physics laboratories and for the large variety of front-end modules (ADC, TDC, QDC, scalers, ...) commercially available.

The main guidelines of the VIPER Data Acquisition System (DAQ) design have been:

- easy integration of homemade electronics with standard commercial ones;
- possibility of low cost upgrading;
- expandability toward multiprocessor and multi-node configurations;
- utilization of network distributed resources;
- reduced CPU obsolescence problems;
- flexible software architecture to simplify module set-up and on-line monitoring;
- high transportability of the system to different laboratories.

Fig. 2.8 is a picture of the front-end VME crate used during the experiments, while Fig. 2.5 shows a schematic diagram of the electronic circuit with the hardware/software components developed in this project drawn in orange. The board containing the silicon detector, the ASIC chip and the related electronics is interfaced to the VME bus through the ASIC to VME Interface (AVI) board. There is one AVI board per chip. The Trigger Supervisor Interface (TSI) board is a highly versatile board and is the trigger filter and deliverer for the whole front-end.
2.2. Readout electronics

Figure 2.8: The VIPER front-end VME crate containing, from left to right, the VME-PCI bridge board (gray), a commercial 32-channel ADC (green), 16 AVI boards (yellow) and the TSI board.

ASIC to VME Interface (AVI) board

The AVI board manages the communication links between the chip and the VME bus for operations like sending setup or calibration commands to the chip. When the acquisition is running and the TSI board asserts a valid trigger signal, the AVI sends the trigger command to the chip and makes available to the VME bus the data stream coming from the chip. Signals regarding the internal status of the AVI board and/or the connected chip are sent back, via the control bus, to the TSI board which, in turn, has also the role of reporting occurrences of errors, FIFO half-full signals, and other important control signals.

Trigger Supervisor Interface (TSI) board

The TSI board provides a logical combination (OR/AND) of all the input channels devoted to generate a trigger signal. When a channel proposes a trigger signal, before promoting it to a valid trigger, the TSI board controls that the whole system is ready (i.e. no AVI is busy, no error occurred, acquisition is running, ...). It is possible to mask or sample one or more of the input channels and to force a valid trigger assertion for testing purposes. An external inhibit signal can also be used to integrate commercial boards in the readout system. This is the case for the ADC used to encode the energy signal from the rear side of the silicon detector: the inhibit signal comes from
the ADC, and is kept asserted by the ADC itself until the data digitalization cycle has been completed.

To evaluate the dead time, the TSI board has been equipped with rate meters and counters for the proposed and accepted triggers. Furthermore, dividers by 10/100/1000 can also be used for the input channels. Finally the TSI board is also used to synchronize all the front-end boards in order to guarantee a common time stamp.

To ensure a complete reconfiguration of the developed electronics and flexibility with respect to wider experimental requirements, Full-Programmable-Gate-Arrays (FPGAs) have been extensively used in the design of the boards along with advanced techniques to optimize the connection arrangement between them.

**Interrupt generator board**

In the set-up used at RIKEN, a new module, called Interrupt generator board, has been inserted in the VME bus. The module is connected to the TSI and avoids the DAQ polling into the TSI registers to recognize the arrival of a valid trigger signal. In the new configuration, the trigger generation is directly communicated from the Interrupt to the DAQ, reducing the time spent by the acquisition system for this operation with a consequent lowering of the DAQ dead time.

**2.2.4 Data Acquisition System**

The VME bus is, in the present set-up, connected to a PC via a commercial VME-PCI bridge. This solution reduces the problems due to the CPU obsolescence, allows a progressive low-cost enhancement of the DAQ system performances with the upgrade of the CPUs available on the market, and does not link the DAQ software to a specific platform. The choice of transferring the CPU from the VME bus to the PC also allows taking the most advantages from the continuous improvement of the I/O devices (disks, RAM, DVD writer, ...) of the PC industry.

The whole system, schematically shown in Fig. 2.9, is accessible over the network because of the **Server/Client** architecture. The **Server** is the process that manages the hardware and runs on the PC directly linked to the front-end. The **Clients** operate on the front-end and monitor the data acquisition through the **Server**. The DAQ system can manage more than one VME crate and includes an innovative and general method to set-up the front-end modules, and the relative on-line analysis, which eliminates the burden due to implementation of software drivers for new modules. The user
2.3 Geometrical efficiency

Due to the non-spherical geometry of the EXODET array, each strip covers a wide range of polar angles $\theta$. For this reason, a Monte-Carlo simulation of the whole apparatus is needed to estimate the solid angle coverage of the EXODET pixel-structure, the polar angles $\theta$ and the solid angle $\Delta \Omega_s$ subtended by each $\Delta E$ strip. Appendix B will present one of the macros used to perform the simulation.

As we have seen in Figs. 2.1 and 2.2, the EXODET telescopes are arranged in two cubic boxes around the beam direction: one at forward angles and the other in backward direction. Two cubes form each box: the inner one (with an edge of 51.50 mm) with the $\Delta E$ detectors and the outer one (with edge of 62.90 mm) with the $E$ layers. The target holder is placed in the middle of the two boxes and the distance between the target plane and the
closest detector edge (active area) is 2.5 (4.0) mm, see Fig. 2.10 for additional geometrical details. It is important to underline that the EXODET configuration could be easily modified according to different experimental set-ups and different physical purposes. In the present case, we refer to the original configuration used during the experiment performed at ANL (see Sec. 3.3), where the array geometry was completely symmetric with respect to the target position. We also remind that the strips of the ΔE layer are mounted perpendicularly to the beam direction and their number increases going away from the target both in forward and backward direction. In agreement with this assumption, strip no. 1 is the closest to the target plane and strip no. 100 is the furthest one. For the E layer the situation is different, since these strips are parallel to the beam direction. The strips are clockwise labeled with respect to the beam direction for the detectors at forward angles and anticlockwise for those at backward angles.

Fig. 2.11 shows the solid angle ΔΩa covered by each ΔE strip. The solid angles have been evaluated assuming a random emission both in polar (0 ≤ θ ≤ π) and azimuthal (0 ≤ φ ≤ 2π) angular ranges from an ideal point-like source located at the intersection between the beam direction and the
2.3. Geometrical efficiency

Figure 2.11: Solid angle coverage, \(\Delta \Omega_s\), of the EXODET \(\Delta E\) strips, estimated via Monte-Carlo simulation.

target plane. Despite the non-spherical shape of EXODET, it is still possible to appreciate a sinusoidal trend, i.e. the \(\Delta \Omega_s\) distribution follows quite well the ideal behavior \(d\Omega = \sin \theta d\theta d\phi\). This trend is clearer for the first strips, where the array is more similar to a 4\(\pi\)-geometry detector since the strips are closer to the target. On the other side, the trend is rather flat for the last strips, which are the furthest ones, and so the solid angle coverage results to be almost the same.

The Monte-Carlo code also allows to estimate the ranges of the polar angles \(\theta\) covered by each \(\Delta E\) strip. Fig. 2.12 shows an example for three groups of strips of the forward hemisphere using a \(\Delta \theta\) binning of 1°. The values have been obtained considering only the \(\Delta E\) strip-structure and forgetting the geometrical constraints imposed by the rear \(E\) detector and by its own strip-structure. Upper panel (Fig. 2.12a) shows the behavior of the strips near the target position, the middle one (b) that of the intermediate strips and the lowest one (c) that of the strips close to the further edge of the detector.

Each strip subtends a wide range of polar angles \(\theta\). The width of this range gets larger and larger as the strip number increases. For the first strips, the whole range is restricted to 5-7 degrees and there is always a main contribution coming from a single polar angle \(\theta\). For the middle strips the range is wider (around 10-12 degrees), but also in this case it is still possible to distinguish in all the distributions a quite sharp peak, which accounts for more than half of the strip solid angle coverage. For the last strips, the range is as large as for the previous case (11-12 degrees), but the distributions are
much flatter and, except a few cases, all the subtended angles $\theta$ practically contribute in the same way to the solid angle covered by the strip. However, we saw that for most of the strips the main contribution usually originates from a very limited number of $\theta$ angles. This information is very useful to reconstruct the $\theta$ angular distribution of particles stopping in the $\Delta E$, even if the position information of $E$ is not present. In this case we can associate to each strip an average $\theta$-value, within an indetermination of 1-3 degrees, depending on the distance between the target and the considered strip. This is, for example, the case of $^{17}$F scattering from a $^{208}$Pb in the energy range around the Coulomb barrier.

Using the simulation we can also estimate the total solid angle coverage of the whole EXODET array. In order to calculate the geometrical efficiency of the non-segmented rear sides of the detectors, we did not initially consider the pixel-structure of the telescope. The obtained values were: $9.0322 \pm 0.0002$ sr (corresponding to a 71.9% coverage of $4\pi$ sr) for the sum of the 8 $\Delta E$ detectors, $7.8201 \pm 0.0002$ sr (62.2% of $4\pi$ sr) considering only the 8 $E$ layers and $7.5711 \pm 0.0002$ sr (60.2%) for the 8 two-stage telescopes. The
geometrical efficiency of a whole telescope is slightly lower than that of a single $E$ detector, since the requirement of a coincidence with the first layer introduces a shadow on the surface of the second stage.

To evaluate the geometrical efficiency of the EXODET pixel-structure, we have to take into account also the dead layers between two neighboring strips. Since the strip pitch is 500 $\mu$m, the 50 $\mu$m dead layer decreases the overall efficiency of about 10% for each stage. In fact the outcoming efficiency are: $8.1289 \pm 0.0002$ sr (64.7% of $4\pi$ sr), $7.0381 \pm 0.0002$ sr (56.0%) and $6.1354 \pm 0.0002$ sr (48.8%) for the $\Delta E$ cubes, for all the $E$ detectors and, respectively, for the whole EXODET array. Table 2.2 summarizes all the efficiencies reported up to now.

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<tr>
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<td>$9.0322$ sr (71.9%)</td>
<td>$8.1289$ sr (64.7%)</td>
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<tr>
<td>$E$ detectors</td>
<td>$7.8201$ sr (62.2%)</td>
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<td>$\Delta E - E$ telescopes</td>
<td>$7.5713$ sr (60.2%)</td>
<td>$6.1354$ sr (48.8%)</td>
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</table>

Table 2.2: Geometrical efficiencies for the two detector stages which form the EXODET apparatus and for the whole array. The column labeled “rear side” refers to the geometrical efficiencies obtained for the non-segmented rear sides of the considered detectors, whereas the other one takes into account also the front sides, which are divided into 100 strips with a dead layer of 50 $\mu$m between them.

The errors have been estimated with the formula usually used for the error evaluation in Monte-Carlo simulations [Eva00]

$$
\sigma_{\Delta \Omega} = \sqrt{\frac{\Delta \Omega}{N} (4\pi - \Delta \Omega)}
$$

(2.1)

where $N$ is number of events generated for the simulation (in our case $N = 1.0 \times 10^9$).

Fig. 2.13 shows the comparison between an ideal $4\pi$-geometry detector array and the two EXODET geometrical efficiencies as a function of the polar angle $\theta$. The upper curve refers to the $\Delta E$ strip-structure efficiency, while the lower one to the pixel-structure. The narrow hole around 90° corresponds to the efficiency lost cause of the target support. We can also appreciate the angular ranges not covered by the two stages: from 0° to 25° (30°) at forward angles and from 155° (150°) up to 180° for the $\Delta E$ layer (for the whole telescopes). This difference is due to the fact that the detectors of the same telescope are mounted at the same distance from the target plane.
Figure 2.13: Comparison between the solid angle covered by an ideal $4\pi$-geometry detector, the $\Delta E$ strip-structure and the $\Delta E - E$ pixel-structure of EXODET. With the symbol $\Delta \Omega_\theta$ we mean the ratio $\frac{\Delta \Omega}{\Delta \theta}$ calculated for $\Delta \theta$ binning of 1°.

and the distance between them is 5.2 mm. So their ranges of covered polar angles $\theta$ are slightly different, as already seen also for the total solid angle coverage, as a consequence particles with high tilted trajectory can only be detected by the $\Delta E$ stage. In the intermediate ranges, there is a reduction of $\sim 10\%$ from an ideal $4\pi$ geometry detector and our $\Delta E$ efficiency. This originates from the 50 $\mu$m-dead layer between two adjacent strips of the $\Delta E$ detector. The segmentation of the $E$ layer introduces an additional reduction of $\sim 10\%$ when we move from the $\Delta E$ stage to the $\Delta E - E$ pixel-structure. These effects are clearly visible in Fig. 2.13.

2.4 Geometrical efficiency with RIB’s

Due to its large solid angle coverage, the EXODET array is well suited for studying nuclear reactions induced by low intensity beams. On the other side, its high granularity allows the kinematics reconstruction of the detected events. In particular, in the Coulomb barrier energy range, knowledge of momentum and energy of the reaction fragments are essential to really understand the reaction mechanism.

As already pointed out in the Introduction, RIB’s delivered up to now
by first generation RIB's facilities also suffer of poor energy and position resolutions. To overcome these problems energy and position of the incoming beam particles are measured with appropriate detectors placed inside the beam line. For example, in the case of the RIPS beam-line at RIKEN, the incident energy is event-by-event reconstructed via Time-of-Flight (ToF) measurements over $\sim 5$ m and the beam position on the target is determined with two Parallel-Plate-Avalanche-Counters (PPACs) placed just in front of the scattering chamber. In this way, the powerful EXODET pixel-structure together with an event-by-event knowledge of beam energy and position on the target allows the scattered particle polar angle ($\theta$) reconstruction.

A Monte-Carlo simulation has been used to test the capabilities of our experimental apparatus for such poor position resolution beams. Starting from the code previously discussed, we moved from a point-like source to a diffuse source with a bi-dimensional Gaussian distribution around the target central position. We used two configurations: the first with a Full-Width-Half-Maximum (FWHM) of 1.0 cm and the other with 2.0 cm-FWHM.

![Figure 2.14: Comparison between the solid angle coverages $\Delta \Omega_s$ of the EXODET $\Delta E$ strips with an ideal point-like source and two bi-dimensional Gaussian distributions on the target: the first one with a FWHM = 1.0 cm and the other with FWHM = 2.0 cm.](image)

Fig. 2.14 shows the comparison between the solid angle coverage calculated in the previous Section with those obtained with two large beam-spots on the target. The position spread on the target increases the solid angle covered by the first 15 strips, where the effects of the reduced distance between the detector and the target hit position are larger. For the intermediate strips there is a sizeable efficiency decrease, whereas for the furthest
strips the larger beam-spot only slightly decreases the solid angle coverage \( \Delta \Omega_n \). Fig. 2.14 also shows that the overall effects get larger and larger as the FWHM of the incoming particle distribution increases.

Fig. 2.15 shows the ranges of the polar angles \( \theta \) covered by three groups of EXODET \( \Delta E \) strips for the two considered beam-spots: panels a), b) and c) for the Gaussian distribution with FWHM = 1.0 cm and panels d), e) and f) for that with 2.0 cm-FWHM. Also in these plots a \( \Delta \theta \) binning of 1° has been adopted. In these cases each strip subtends a much wider range of \( \theta \) angles than with an ideal point-like source: in the best case (strip no. 1 and FWHM = 1.0 cm) we have at least 15 angles. However, all the distributions show a bell shape, but the main contribution to the total strip solid angle coverage does not originate anymore from a few (one or at most two) polar angles like in the previous case. In fact, for a beam-spot of 1.0 cm, the widths of the distributions range from 7-8 degrees for the strips closest to the target up to 15-16 degrees for the furthest ones. When the beam-spot is 2.0 cm, the distributions spread into still wider ranges: a single strip can cover more than 40 polar angles and the width of the bells can easily reach more than 20 degrees.

Monte-Carlo simulations clearly demonstrate that with a poor position resolution beam, it is quite difficult to reconstruct the \( \theta \) angular distribution of particles stopped in the \( \Delta E \) layer. In fact the angular range covered by any strip is too wide to associate to each strip an average \( \theta \)-value with a small uncertainty. For this reason, the position information of the \( E \) detector together with the knowledge of the position hit on the target are mandatory requirements to disentangle the actual scattering angle among all those covered by the \( \Delta E \) strip. Therefore the thickness of the \( \Delta E \) layer has to be chosen in order to guarantee that the scattered particles do not stop in the first layer and also the beam-line has to be equipped with a tagging system of the incoming particles.

Last, but not least, Monte-Carlo simulations also allow the evaluation of the total solid angle coverage of EXODET with the two considered beam-spots. As we can deduce from the comparison between Tables 2.2 and 2.3, the geometrical efficiencies of EXODET only slightly decrease with respect to the case of an ideal point-like source. The overall change is less than 0.5% for the \( E \) layer, which is further from the target region, whereas for the \( \Delta E \) and for the whole array efficiencies there are minor changes (\( \sim 0.2-0.3\% \)) only in the case of 1.0 cm-beam spot. When the spread gets larger, particles can hit target positions very close to the inner detectors reducing the geometrical efficiency of \( \sim 1.0\% \) both for the \( \Delta E \) layer and the whole telescope. Anyhow this efficiency reduction is quite small and the total solid angle coverage is still very large even with a wide beam spread.
Figure 2.15: Ranges of polar angle coverage for three groups of EXODET $\Delta E$ strips and for two bi-dimensional Gaussian distributions on the target: panels a), b) and c) with FWHM = 1.0 cm and d), e) and f) with FWHM = 2.0 cm. Data have been discretized with a bin width of $\Delta \theta = 1^\circ$.

The comparison between Figs. 2.13 and 2.16 shows that the geometrical efficiency is practically beam-spot independent in the intermediate ranges.
Figure 2.16: Comparison between the solid angle covered by an ideal 4π-geometry detector, the ΔE strip-structure and the ΔE − E pixel structure of EXODET for two bi-dimensional Gaussian beam-spots on the target.

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<td>8.0923 sr (64.4%)</td>
<td>7.9750 sr (63.5%)</td>
<td></td>
</tr>
<tr>
<td>E detectors</td>
<td>7.0255 sr (55.9%)</td>
<td>6.9842 sr (55.6%)</td>
<td></td>
</tr>
<tr>
<td>ΔE − E telescopes</td>
<td>6.1068 sr (48.6%)</td>
<td>6.0105 sr (47.8%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: Geometrical efficiencies for the two detectors which form each telescope of EXODET and for the whole array. Upper (lower) panel shows the results of the simulation for the continuous rear (segmented front) side of the detectors with two bi-dimensional Gaussian distributions on the target. The uncertainty of each value, estimated with the formula 2.1, is 0.0002 sr.

(θ between 45° and 70° and from 110° up to 135°) both for the ΔE strip-structure and the ΔE − E pixel-structure. In these regions, even if the beam diameter deeply modifies the ranges of polar angles covered by any strip and each θ angle is smeared over wider number of strips, the global effect of a large beam-spot vanishes when we consider all the detector strips. Fig. 2.16
clearly illustrates that the change of the beam size only affects the tails of the distribution, namely the region around the target ($\theta \sim 90^\circ$) and the two poles ($\theta \leq 25^\circ$ for the forward hemisphere and $\theta \geq 155^\circ$ at backward angles). In these ranges the shapes of the distributions are typically smoother than in the ideal case, since we are dealing with a diffuse source. Moreover, this feature increases the geometrical efficiency at very small and large $\theta$ angles, even if with low intensity, and decreases the efficiency around the target.

We can eventually conclude that Monte-Carlo simulations confirm that the EXODET apparatus is well suited for studying nuclear reactions induced by low intensity RIB’s with poor position resolution. In fact, its very large solid angle coverage and its high granularity ensures the detection of charge particles passing through the first layer with high geometrical efficiency and small position indetermination.
Chapter 3

Experiment I: $^{17}\text{F} + ^{208}\text{Pb}$

3.1 Secondary beam production: ISOL and In-Flight methods

Short-lived RIB’s are currently produced either via Isotope-Separation-On Line (ISOL) method or via In-Flight (IF) technique. A driven accelerator provides the particles inducing nuclear reactions in a production target.

In the ISOL method the primary beam is usually a very light particle beam ($p$, $d$ or $n$) impinging on a high-$Z$ thick target, where it is fully stopped. The radioactive products are transported in an ion source to be further ionized and later re-accelerated. The outcoming beams are optically (emittance, energy resolution, timing structure) of excellent quality but the thermalization and re-ionization processes can lead to large losses for short-living nuclei or for isotopes from refractory elements.

In the IF method primary particles have high-$Z$ and energy (tens or hundreds of MeV per nucleon) and the target is thin enough to ensure the resulting fragments to recoil out of the target itself. This technique is applicable to very short-living nuclei (half-lives up to $\sim \mu s$), since it does not depend on the chemical properties of the produced nuclei and only the time-of-flight from the primary target to the reaction chamber induces decay losses. An advantage of the IF method with respect to ISOL is that it allows for an easy variation of the energy of the reaction products within a certain range and can be implemented in existing facilities. The radioactive beams are produced already at high energy and the goal is to select the right isotope and to transport it to the experimental set-up. These advantages come at the price of a lower beam quality. In fact, these secondary beams sometimes need to be slowed down in degrader foils to match the required reaction energy range with a consequent poor energy resolution.
3.2 $^{17}\text{F}$ beam production at ANL

In our experiment, the $^{17}\text{F}$ radioactive beam ($T_{1/2} = 64.5$ s) was produced at the Argonne National Laboratory (ANL, Illinois, USA) via the IF technique. General requirements for a "good" secondary beam suitable for nuclear physics experiments are: high beam current, small beam spot on the target, small angular and energy spread. For the IF technique, all this translates into several constrains for the choice of the primary reaction, the design of the production target and the layout of the beam transport system:

1. inverse reactions, *i.e.* to bombard a light-$Z$ target with an heavier-$Z$ projectile, via for example $(p, n)$, $(d, n)$ or (He,$d$) reactions, preferably with negative $Q$-values, should be preferred since the reaction products are emitted in narrow cones at forward angles in the laboratory frame. Moreover, reactions with forward-peaked differential cross sections should be chosen;

2. the angle and energy acceptance for the beam transport system should be as large as possible. Typical values are a few degrees with respect to the primary beam direction for the opening angles and several MeV for the energy spread;

3. the primary beam spot on the production target should be small;

4. the production target should stand the largest possible beam current, introduce a small angle-straggling and have a high isotopic concentration in its active area;

5. passive layers (*i.e.* windows) of gas production targets, in particular on the exit side, should be as thin as possible;

6. a highly efficient mechanism for suppressing the intense primary beam passing through the production target should be provided;

7. the $\gamma$, $x$-ray and neutron radiation produced in the primary target, potentially interfering with the experimental equipments, should be shielded;

8. the low intensity secondary beam should be high efficiently selected and transported from the production target to the reaction chamber.

In our case, the inverse reaction $p(^{17}\text{O}, ^{17}\text{F})n$ ($Q$-value = -3.54 MeV) was used [Har00]. A high intensity ($\geq 250$ pA) 102-MeV $^{17}\text{O}$ beam, provided by the ATLAS superconducting linear accelerator [Bol93], impinged on a gas
cell filled with hydrogen at a pressure of \( \sim 600 \) Torr. Because of the inverse kinematics of the reaction, \(^{17}\text{F}\) ions were emitted within a narrow cone (\( \sim 2-4^\circ \)) in the forward direction. The \(^{17}\text{F}\) energy, measured with an Enge-split pole magnetic spectrograph [Eng64], was 90.4 MeV with a FWHM of 1.4 MeV (corresponding to a resolution of \( \sim 1.5\% \)) and the intensity was \((1-2 \times 10^3)\) pps. The beam diameter was \( \sim 5 \) mm at the target position (around 15 m downstream the production gas-target). The main contaminant in the secondary beam was the \(^{17}\text{O}\) primary beam, with the same magnetic rigidity as \(^{17}\text{F}\), but with an energy \( \sim (8/9)^2 \cdot E_{^{17}\text{F}} \). As we will see in the following the intensities of these two beams were of the same order of magnitude.

### 3.2.1 \(^{17}\text{F}\) production gas target

![Schematic cross-section of the gas target used during the \(^{17}\text{F}\) experiment.](image)

Since no foil target with H-rich compounds can support the high primary beam current, the use of a gas target was adopted. The target cell (see Fig. 3.1 for a simplified cross-section of the gas target used during the experiment) consisted of a double-walled cylinder with 2.54 cm inner diameter and the length of 7.5 cm [Har00]. The volume between the walls was filled with a constantly flowing liquid nitrogen. Each window consisted of 1.9-mg/cm\(^2\)-thick HAVAR foil, soldered to a stainless steel ring with an inner diameter of 1.3 cm. This ring was mounted on the gas cell using an indium gasket.

The life-time of the HAVAR foils depends on the beam spot and on the degree of cooling and it consequently limits the maximum current of the primary beam. Since it is not fruitful to defocus the \(^{17}\text{O}\) beam because of the transport efficiency, cooling the cells to liquid nitrogen temperature both
improves the life-time of the windows and provides a higher density target at same pressure. In order to reduce the time needed for opening the beam line after a window failure, a stack of three gas cells was used.

3.2.2 Beam-line configuration

![Diagram of beam-line configuration](image)

Figure 3.2: Displacement of the two ATLAS superconducting resonators, the gas target, the refocusing solenoid and the bending magnet along the beam line. The envelopes of the $^{17}$O primary and $^{17}$F secondary beams are also shown in the drawing. Horizontal scale not valid.

The gas target was placed between two ATLAS superconducting resonators, in front of a 2.2 T superconducting solenoid, and it could be moved over a distance of 0.63 m along the beam line. This configuration is a compromise between the sufficiently small secondary beam envelope required by the collimator of the second resonator and the largest ion divergence that could be refocused by the optical elements in the beam line. The “bunching” resonator located 10 m upstream the production target provided the focusing of the primary beam at the gas cell and minimized the longitudinal emittance of the $^{17}$F beam. The superconducting solenoid was installed just behind the gas target to increase the angular acceptance, while the second resonator was employed to reduce the energy spread of the secondary beam. Fig. 3.2 shows the displacement of the two resonators, the gas target and the refocusing solenoid along the beam line together with the envelopes of the $^{17}$O primary and $^{17}$F secondary beams.

For this experiment we used the ANL spectrograph beam line. The $^{17}$F$^+$ ions were separated from the $^{17}$O primary beam in a 22° bending magnet and then refocused into a beam spot of $\sim 0.5$ cm by two quadrupole doublets (see
3.3. Experimental set-up

In the first experiment with the EXODET apparatus only one section of the whole array was used. A two-stage telescope was placed in the backward direction and another one was located at forward angles. The distance between the detector active area and the target plane was 4 mm for both telescopes (see Fig. 2.10 for the geometrical details of the set-up). The thicknesses of the detectors were: 70 µm for the ΔE layers and 500 µm for the E stages. The forward telescope was used for data normalization since at forward angles we expected to have purely Rutherford cross section in the energy range of the experiment. The target was a self supporting 1-mg/cm²-thick $^{208}$Pb foil.

A Monte-Carlo simulation was performed to estimate the total solid angle covered by the array and the ranges of the polar angles $\theta$ covered by the two telescopes for the present set-up geometry. Since the beam spot at the target position was rather small (FWHM $\sim 0.5$ cm), the results differ less than 0.2% from those obtained for an ideal point-like source. In fact the solid angle

Figure 3.3: Schematic view of the optical elements that compose the spectrograph beam-line. $T_1$ indicates the production target, while SOL is the re-focusing solenoid and RES is the second superconducting resonator, already reported in Fig. 3.2.

Fig. 3.3). The total efficiency of the beam transport system was measured to be at maximum 2.5%. The incident energy of the $^{17}$F ions was regularly measured during the experiment with the Enge-split pole magnetic spectrograph [Eng64] positioned at $0^\circ$, after having reduced the energy of the $^{17}$O primary beam impinging on the gas target to a sufficient low intensity.
coverage of the $\Delta E$ front side of each telescope resulted to be 1015.4 sr as in the ideal case. The polar angle ranges from 22$^\circ$ up to 84$^\circ$ and from 96$^\circ$ up to 158$^\circ$ were subtended. These ranges are a bit larger than for a point-like source, however the additional angles have very small solid angles coverages.

During the experiment, the data acquisition system was triggered by logical “OR” of all the energy signals. The collected data were: the energy signals coming from the not-segmented sides of the $\Delta E$ and $E$ detectors, and the position information processed by the ASIC chip: hit strip number, Jitter Time (JT) and Time-over-Threshold (ToT) (see Par. 2.2.2 for additional details about the chip output data stream).

Within the chip time resolution of 67 ns and the consequent low energy resolution, the ToT range is sufficient to separate two particles with different energy domains hitting two different strips of the same $\Delta E$ detectors. This feature is particularly helpful for the selection of the breakup events $^{17}F \rightarrow ^{16}O + p$. In fact, the $^{17}F$ breakup channel produces two particles, a proton and an $^{16}O$ ion, that can hit the same $\Delta E$ detector of the same telescope. Their expected energy ranges (as we will see in the Sec. 3.6.1) were such that the $^{16}O$ ion was stopped in the $\Delta E$ stage and the proton in the second stage. Moreover the proton deposited in one strip a much smaller energy than the one released by the $^{16}O$ in another strip. Therefore, using the ToT information, it is possible to distinguish which of the two strips was hit by the proton and which one was hit by the $^{16}O$ ion.

From the data we determined the $^{17}F$ scattering angular distribution at backward angles and we performed a first direct measurement of the $^{17}F \rightarrow ^{16}O + p$ ($S_p = 0.601$ MeV) exclusive breakup cross section at energies below the Coulomb barrier, with the only limitation of the statistical accuracy.

All the data analysis has been performed with the package Vism [Var89] using Linux as the operating system.

### 3.4 Experimental results

Fig. 3.4 shows a typical energy spectrum collected during the experiment with the $\Delta E$ detector of the backward telescope. Excluding the background signals at very low energy, we can clearly distinguish three broad structures. On the high energy side there are two bumps: the one at higher energies comes from the $^{17}F$ ions scattered by the target, while the other, at lower energy, arises from the $^{17}O$ elastic scattering from $^{208}Pb$. The average energy of the contaminant $^{17}O$ primary beam is around $(8/9)^2$ that of the $^{17}F$ secondary beam, since the two nuclides are transmitted by the bending magnet with the same magnetic rigidity. These bumps are very broad and their
structures mainly arise from the large solid angle coverage, the large range of subtended \( \theta \) polar angles and the energy lost in the target. The overall energy resolution was estimated to be \( \sim 18\% \), essentially originating from the very large kinematic spread (\( \theta \) angles ranging from \( 96^\circ \) to \( 158^\circ \)). However, an energy resolution of 3-4% could be achieved selecting a limited number of strips (5-10, according to statistics). In Fig. 3.4 at low energies, one can see a third peak originated by light particles passing through the first layer, as will be explained in the following section.

The first information that can be used to disentangle all the components of the spectrum is the Jitter Time (JT). We remind (see Sec. 2.2 for additional details) that the JT, being the time between the arrival of a signal to the chip and the assertion of a valid trigger, is a sort of time-correlation measurement. Fig. 3.5 shows the JT spectrum of all the particles hitting the \( \Delta E \) detector. We clearly see that the spectrum is sharply-peaked around channel 10 (it means that, considering the time resolution currently available with the used chip, the actual Jitter Time of the signals is peaked around 667 ns). The presence of this very narrow peak indicates the time-correlation of most of the recorded events and confirms that this tool is very useful to avoid spurious and/or uncorrelated signals. Fig. 3.5 shows that there are also smaller contributions arising from the channels close to the correlation peak,
Figure 3.5: Jitter Time (JT) spectrum of all the data collected with the backward ΔE detector. The distribution is sharply peaked around channel 10, corresponding to a correlation time of 667 ns (10 x 67 ns).

Figure 3.6: Time over Threshold (ToT) spectrum of all the data collected with the ΔE detector.

...in particular from the channels 9 and 11. In order to avoid any impurity coming from possible uncorrelated events in the tails of the JT distribution,
in the following analysis only the events with JT equal 10 will be considered. In a particular way the events corresponding to JT = 9 were not included since a larger noise would have been introduced at low energies.

![Graph](image)

**Figure 3.7:** Time over Threshold (ToT) spectra of the events detected by the ΔE detector with different gates on the Jitter Time (JT) and on the energy lost in the ΔE layer. ToT spectra correspond to the following gates: (a) JT = 10 and ΔE ≥ 59 MeV (corresponding to the $^{17}$F scattering peak); (b) JT = 10 and $45 \leq \Delta E \leq 59$ MeV (events belonging to the $^{17}$O elastic scattering peak and (c) JT = 10 and $\Delta E \leq 10$ MeV (light particle range).

The spectrum of the Time-over-Threshold (ToT), the other digital information provided by the readout chip, is shown in Fig. 3.6. We remind that ToT is the time spent by a signal over an externally settable threshold in the ASIC chip [Par99] and it gives a rough information about the energy lost by a particle in a strip. Three different domains can be easily distinguished in the ToT spectrum. At high ToT-values only two channels have significant population, ToT = 6 and ToT = 8, whereas at low ToT-values all the events are smeared over 3 channels (ToT = 2-4). Because of the limited time-resolution of the chip, it is impossible to appreciate any underlying structure of the
peaks. The nature of these three regions can be easily understood selecting different energy gates in the $\Delta E$ spectrum.

![Delta E spectra](image)

Figure 3.8: $\Delta E$ spectra corresponding to different gates on $JT$ and $ToT$ parameters for the events detected with the $\Delta E$ layer of the backward telescope. Upper panel: $JT = 10$ and $ToT = 6$; middle panel: $JT = 10$ and $ToT = 8$; bottom panel: $JT = 10$ and $ToT = 2-4$.

Fig. 3.7 shows the results of these selections for the events with $JT = 10$. In the upper panel (a) we can see the ToT spectrum for the scattered $^{17}F$ scattered ions, in the middle one (b) we have the ToT spectrum for the events in the $^{17}O$ energy domain and in the bottom one (c) the spectrum for the light particles energy range. We clearly see that the $^{17}F$ scattering events have a ToT sharply-peaked around 400 ns ($6 \times 67$ ns), the $^{17}O$ ions correspond to a ToT-value of 533 ns ($8 \times 67$ ns), whereas this parameter is smaller than 266 ns ($4 \times 67$ ns) for light particles. At the contrary to a first inspect, the ToT associated to the Oxygen ions is higher than that of the $^{17}F$ ions. This is most probably due to a difference in the pulse shapes between the two ions and still needs to be further investigated. However, since the two regions are well separated and no ambiguity is possible between the two
ion species, this finding does not affect at all our results.

Let us now move back to the $\Delta E$ spectrum to see the effects of both JT and ToT selections on the energy signals. Fig. 3.8a shows the $\Delta E$ spectrum gated by JT = 10 and ToT = 6: practically only the $^{17}\text{F}$ scattering peak survives these gates. Fig. 3.8b shows the same $\Delta E$ spectrum, but gated by ToT = 8 instead of ToT = 6: in this case only the $^{17}\text{O}$ elastic peak populates the resulting spectrum. Lower panel (Fig. 3.8c) represents the $\Delta E$ spectrum gated by the lowest ToT-values (2-4) and JT = 10. Here only the low energy events, that is those passing through the first stage of the telescope, are present. This is a clear indication of the capabilities given by the digital information processing performed with the ASIC chip.

3.5 $^{17}\text{F}$ scattering event analysis

Since the $\Delta E$ detectors used for this experiment were 70 $\mu$m thick and the beam energy was 90.4 MeV, all $^{17}\text{F}$ nuclei were stopped in the first layer (see Sec. 2.1), making a $\Delta E - E$ pixel analysis impossible. For this reason, in the analysis of the $^{17}\text{F}$ scattering from a $^{208}\text{Pb}$ target, all the events in the Fluorine peak with only one $\Delta E$ hit strip, with JT = 10 and with ToT = 6, were considered.

Our experimental resolution did not allow to separate the contributions coming from the first excited states both in $^{17}\text{F}$ (at 0.4953 MeV) and in $^{208}\text{Pb}$ (at 2.614 MeV). However, from Distorted Wave Born Approximation (DWBA) calculation, as we will see in more detail in Sec. 5.1, these contributions are predicted to be very small in comparison with the pure elastic cross section.

Fig. 3.9 shows the strip count numbers $N_s$ for each strip of the $\Delta E$ detector placed at backward angles. We clearly see that the distribution shows a maximum around strip no. 20. This is a direct consequence of the target screening. In fact, since the target used was rather thick and the distance between the detector active area and the target frame was only 4 mm, the $^{17}\text{F}$ scattered ions with angles close to 90° could hit the first strips only after passing through a much larger target thickness and consequently after losing a larger amount of their incident energy. The corresponding $\Delta E$ signals could be out of the identified Fluorine peak and also the digital ToT information could be out of the range previously defined for the $^{17}\text{F}$ ions. However, this fact did not influence our results, since the first 15 strips were removed from the off-line analysis. The only consequence was that the $^{17}\text{F}$ scattering differential cross section could be evaluated starting from $\theta = 115°$.

Due to the non-spherical shape of the detector, a Monte-Carlo simulation
(see Secs. 2.3 and 2.4) was necessary to reconstruct the behavior of the scattering differential cross section starting from the strip count numbers \( N_s \). The parameters calculated via the Monte-Carlo method to determine the \(^{17}\text{F}\) scattering angular distribution \( \frac{d\sigma}{d\Omega}(\theta) \) were the following:

\( \theta \), the range of polar angles covered by each \( \Delta E \) strip \( s \) of the backward detector (see Fig. 2.12);

\( \Delta\Omega_s \), the solid angle coverage of each \( \Delta E \) strip \( s \) at backward angles (see Fig. 2.11);

\( \Delta\Omega_\theta \), the solid angle subtended by any polar angle \( \theta \) (with \( \Delta\theta = 1^\circ \)) for the segmented front side of the \( \Delta E \) detector in the backward direction (see Fig. 2.13);

\( \Delta\Omega_{\theta_s} \), the solid angle covered by each polar angle \( \theta \) (with \( \Delta\theta = 1^\circ \)) inside any \( \Delta E \) strip of the backward detector. These quantities are basically the elements of a matrix with 90 lines (one for each backward polar angle \( \theta \) \( \times \) 100 columns (one for each \( \Delta E \) strip). Adding over the columns we obtain what we called \( \Delta\Omega_\theta \), while adding over the lines we get the detector solid angle covered by each \( \Delta E \) strip, namely \( \Delta\Omega_s \);

\( \Delta\Omega_{\theta_F} \), the solid angle subtended by any polar angle \( \theta_F \) for the continuous rear side \( \Delta E \) detector at forward angles, since only the non-segmented
backplane of this detector was used.

The $^{17}$F scattering angular distribution can be now directly reconstructed in the laboratory frame for all the polar angles $\theta$ subtended by the backward detector starting from the counts of each strip $(N_s)$, using the formula:

$$
\frac{d\sigma}{d\Omega}(\theta) = \sum_s N_s \left( \frac{\Delta \Omega_{\theta_s}}{\Delta \Omega_{\theta}} \right) \times \frac{\sum_{\theta_F} \left( \frac{d\sigma_F}{d\Omega} (\theta_F) \right) \Delta \Omega_{\theta_F}}{N_F}
$$

(3.1)

Figure 3.10: $^{17}$F + $^{208}$Pb scattering angular distribution at 90.4 MeV energy. The quoted errors take into account only the statistical uncertainties and those arising from the Monte-Carlo simulation. The continuous line is the result of the optical model best-fit of the angular distribution and will be discussed in Ch. 5.

In the first factor of 3.1 we added the strip count numbers $N_s$ over all the strips $s$ containing the considered polar angle $\theta$. All the addenda have been weighted by the ratio $\frac{\Delta \Omega_{\theta_s}}{\Delta \Omega_{\theta}}$, that is all the counts of a strip $(N_s)$ have been subdivided among all the polar angles $\theta$ proportionally to the solid angle covered $(\Delta \Omega_{\theta_s})$ by each polar angle $\theta$ inside the considered strip $s$. The second factor is the normalization factor. In the energy range of the experiment, i.e. well below the Coulomb barrier, the cross section at forward angles is expected to be purely Rutherford. The numerator was obtained adding the Rutherford cross section $\frac{d\sigma_F}{d\Omega} (\theta_F)$ over all the polar angles $\theta_F$ covered by the $\Delta E$ detector at forward angles. Each addendum was weighted by the detector solid angle $\Delta \Omega_{\theta_F}$ covered by the corresponding polar angle $\theta_F$. The
outcoming sum was eventually divided by the total number $N_F$ of events collected in the $\Delta E$ forward detector.

Fig. 3.10 and Table 3.1 present the evaluated $^{17}$F scattering differential cross section at 90.4-MeV beam energy. As already said, because of the target thickness and the target frame screening, only the experimental points above 115° could be evaluated. The errors plotted in the figure take into account only the statistical contributions and those originated from the Monte-Carlo simulation (see Formula 2.1). One can see that the differential cross section is rather flat up to $\sim 130°$ and then it slight decreases.

<table>
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<tr>
<th>$\theta_{cm}$</th>
<th>$\frac{d\sigma}{d\Omega}$</th>
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</tbody>
</table>

Table 3.1: Angular distribution for the $^{17}$F scattering process from a $^{208}$Pb target at 90.4 MeV energy. The quoted errors consider the statistical uncertainties and those originated from the Monte-Carlo simulation.
3.6 Breakup event analysis

The collected data were also processed to search for breakup events \(^{17}\text{F} \rightarrow ^{16}\text{O} + p\) \((S_p = 0.601\ \text{MeV})\). First of all, a detailed knowledge of the breakup process kinematics is needed to evaluate the possibility to perform such a measurement with our experimental apparatus. In particular, we should calculate:

1. the energy range of the breakup fragments to determine whether they are able to reach the second stage of the EXODET telescopes;

2. the angular spread of the outgoing fragments in order to determine whether only one EXODET telescope covers enough solid angle to detect simultaneously both fragments;

3. the efficiency of the whole EXODET array to perform such a measurement and its dependence on the excitation energy of the breaking nucleus.

As we will see at the end of the discussion, this will be the case where the capabilities of the EXODET array are particularly remarkable.

3.6.1 Breakup reaction kinematics

In our scheme the breakup reaction was described as a sequential process. Under this hypothesis the scattering process from the target can be decoupled from the projectile breakup. In addition, we assumed that the breakup process occurred in the Coulomb field of the target, but far enough from the target to neglect its influence on the projectile fragments, \(i.e.\) no post-acceleration and target recoil effects were considerer. Finally, since the process took place at energy well below the Coulomb barrier, the nuclear interaction neither between the target and projectile nor subsequently between the target and the fragments were considered. This rather simplified description resulted to be a quite realistic approximation of the whole breakup process for the system \(^6\text{Li} + ^{208}\text{Pb}\) in the energy range around the Coulomb barrier [Maz03].

Let us first consider the general case of a scattered particle with mass \(M_P\) breaking into two fragments, labeled 1 and 2, with masses \(m_1\) and \(m_2\), respectively. We will consider later the breakup process \(^{17}\text{F} \rightarrow ^{16}\text{O} + p\) and its relationship with the EXODET array. All the quantities in the Center-of-Mass (CM) system will be reported with the apex (').
In the CM frame the two fragments are emitted with the same momentum along opposite directions and the relationships between their momenta and energies are the following:

$$m_1\vec{v}_1^\prime + m_2\vec{v}_2^\prime = 0$$  \hspace{1cm} (3.2)

$$\frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2 = E_{rel}$$  \hspace{1cm} (3.3)

where $E_{rel}$ is the excitation energy above the breakup threshold $S_x$, that is $E_{rel} = E_x - S_x$, with $E_x$ excitation energy of the breaking nucleus. Replacing Formula 3.2 into 3.3 and using the identity $M_P = m_1 + m_2$, one can easily obtain the absolute value of the two fragment velocities in the CM frame:

$$v_{1,2}' = \sqrt{\frac{2E_{rel}m_{2,1}}{M_P m_{1,2}}}$$  \hspace{1cm} (3.4)

**Energy spread of the breakup fragments**

To calculate the fragment energy $E_i$ (with $i = 1$ or 2) in the laboratory (LAB) system, the velocity vector $\vec{v}_i'$ in the CM frame has to be added to that of the CM ($\vec{v}_{CM}$) as follows:

$$E_i = \frac{1}{2}m_i\vec{v}_i^2$$

$$= \frac{1}{2}m_i(\vec{v}_{CM} + \vec{v}_i')^2$$

$$= \frac{1}{2}m_i(\vec{v}_{CM}^2 + (v_i')^2 + 2\vec{v}_{CM}v_i'\cos\phi).$$  \hspace{1cm} (3.5)

The angle between the two velocity vectors $\phi$ can assume all the values in the range between 0 and $2\pi$. Therefore, the energy of each fragment has a continuum spectrum in the LAB frame and its limits can be easily evaluated by means of the following formula:

$$E_{1,2 \text{ max,min}} = \frac{m_{1,2}}{M_P}E \left(1 + \frac{E_{rel}m_{2,1}}{E m_{1,2}} \pm 2\sqrt{\frac{E_{rel}m_{2,1}}{E m_{1,2}}} \right)$$  \hspace{1cm} (3.6)

where $E$, scattering energy of the breaking nucleus in the LAB frame, is given as a function of the incoming energy $E_0$, the scattering angle in the LAB system $\theta$ and the target mass $M_T$ by the formula [Mar68]:
3.6. Breakup event analysis

\[ E = (E_0 - E_x) \left( \frac{M_P}{M_P + M_T} \right)^2 \left( \cos \theta + \sqrt{\left( \frac{M_T}{M_P} \right)^2 - \sin^2 \theta} \right)^2 \]  

(3.7)

From a first analysis of the formula 3.6, one can see that after a breakup process each fragment carries a fraction of the scattering energy directly proportional to its mass. In our case the average $^{16}$O energy will be about sixteen times larger than that of the proton. Table 3.2 summarizes the energy spread for the two fragments originated from the breakup of a $^{17}$F ion at 90.4-MeV beam energy for selected scattering angles $\theta$ covered by the EXODET array and for $E_{rel} = 0.5$ MeV. This value has been chosen in analogy with the system $^6$Li + $^{208}$Pb, whose excitation function at Coulomb barrier energies shows a maximum at about 0.5 MeV above the $\alpha + d$ breakup threshold [Maz03].

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$E_{^{16}O-max}$ (MeV)</th>
<th>$E_{^{16}O-min}$ (MeV)</th>
<th>$E_{p-max}$ (MeV)</th>
<th>$E_{p-min}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>85.4</td>
<td>79.1</td>
<td>8.7</td>
<td>2.5</td>
</tr>
<tr>
<td>55°</td>
<td>81.4</td>
<td>75.4</td>
<td>8.4</td>
<td>2.3</td>
</tr>
<tr>
<td>80°</td>
<td>76.4</td>
<td>70.5</td>
<td>8.0</td>
<td>2.1</td>
</tr>
<tr>
<td>100°</td>
<td>72.2</td>
<td>66.5</td>
<td>7.7</td>
<td>1.9</td>
</tr>
<tr>
<td>125°</td>
<td>67.7</td>
<td>62.2</td>
<td>7.3</td>
<td>1.8</td>
</tr>
<tr>
<td>150°</td>
<td>64.6</td>
<td>59.2</td>
<td>7.0</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 3.2: Energy spread for the two fragments ($^{16}$O and p) originated from the $^{17}$F breakup at 90.4-MeV beam energy calculated at selected scattering angles $\theta$ and at $E_{rel} = 0.5$ MeV above the breakup threshold ($S_p = 0.601$ MeV).

The comparison between Tables 2.1 and 3.2 clearly shows that the $^{16}$O ion is always stopped in the $\Delta E$ detector of the EXODET telescopes, whereas only low-energy protons are not able to reach the second detector stage. Assuming a squared proton distribution over all the available energy range, only 10% (30%) of the protons cannot pass through the first detector layer for a $^{17}$F scattering angle of the 30° (150°). If the proton energy spectrum has a Gaussian distribution around the average energy value, more than 95% of the protons produced in the $^{17}$F breakup will reach the $E$ detector.
Angular spread of the breakup fragments

In the energy range of our experiment, the module of the CM velocity

$$v_{CM} = \sqrt{\frac{2E}{m_P}}$$

(3.8)

is around one order of magnitude larger than those of the breakup fragments in the CM frame. This implies, as illustrated in Fig. 3.11, that the two fragments are emitted in narrow cones around the scattering direction of the breaking ion. The maximum openings ($\varphi_{i\max}$) of these kinematic cones differs from fragment to fragment and also strongly depends on the projectile excitation energy before the breakup process, as shown by the formula:

$$\varphi_{i\max} = \arcsin \left( \frac{v_i'}{v_{CM}} \right)$$

(3.9)

![Diagram showing kinematic cones of the two breakup fragments](image)

Figure 3.11: *Kinematic cones of the two breakup fragments produced with respect to the scattering direction of the breaking nucleus. If $E_{rel}$ is at least one order of magnitude smaller than the scattering energy and the masses of the produced particles are comparable, the openings of these cones are very narrow ($\varphi_i \leq 10^\circ - 20^\circ$).*

Replacing 3.9 in 3.4, we can evaluate the angular spreads of the fragments 1 and 2 around the direction of the scattered ion:

$$\varphi_{1,2\max} = \arcsin \sqrt{\frac{m_{2,1}E_{rel}}{m_{1,2}E}}$$

(3.10)

Adding this two quantities we can determine the maximum angle $\varphi_{1-2\max}$ between the directions of the two fragments. The comparison of this value
with the granularity of our experimental set-up tells us whether our apparatus is suitable for studying this kind of reaction. Table 3.3 shows the maximum openings of the kinematic cones for a breakup process taking place at 90.4-MeV beam energy and at $^{17}$F scattering angle $\theta = 125^\circ$ (average polar angle of the backward telescope).

<table>
<thead>
<tr>
<th>$E_{\text{rel}}$</th>
<th>$\varphi^{16}_{O,\text{max}}$</th>
<th>$\varphi_{p,\text{max}}$</th>
<th>$\varphi_{p,-^{16},O,\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 MeV</td>
<td>0.9°</td>
<td>14.2°</td>
<td>15.1°</td>
</tr>
<tr>
<td>0.50 MeV</td>
<td>1.2°</td>
<td>20.3°</td>
<td>21.6°</td>
</tr>
<tr>
<td>0.75 MeV</td>
<td>1.5°</td>
<td>25.2°</td>
<td>26.7°</td>
</tr>
<tr>
<td>1.00 MeV</td>
<td>1.7°</td>
<td>29.5°</td>
<td>31.3°</td>
</tr>
<tr>
<td>1.50 MeV</td>
<td>2.1°</td>
<td>37.2°</td>
<td>39.4°</td>
</tr>
<tr>
<td>2.00 MeV</td>
<td>2.5°</td>
<td>44.5°</td>
<td>47.0°</td>
</tr>
<tr>
<td>3.00 MeV</td>
<td>3.1°</td>
<td>59.6°</td>
<td>62.7°</td>
</tr>
</tbody>
</table>

Table 3.3: Kinematic cone openings for the two fragments originated from the breakup process $^{17}$F $\rightarrow^{16}$O $+$ p with respect to the $^{17}$F scattering direction. Calculations have been done at 90.4-MeV beam energy for a $^{17}$F scattering angle $\theta = 125^\circ$. $E_{\text{rel}}$ is the excitation energy above the breakup threshold ($S_p = 0.601$ MeV).

We clearly see that the $^{16}$O ions are emitted in very narrow cones around the $^{17}$F scattering direction, whereas the openings of the proton kinematic cones are much wider, since their velocities in the CM frame are sixteen times larger than those of the $^{16}$O nuclei (see formula 3.4). Table 3.3 also shows the effect of the excitation energy. The openings of the kinematic cones get larger and larger as the excitation energy of the breaking nucleus increases. However, if $^{17}$F excitation energy distribution is concentrated in the interval up to 1 MeV above the breakup threshold as for the system $^6\text{Li} + ^{208}\text{Pb}$ at Coulomb barrier energies [Maz03] and for other light RIB’s at much higher beam energies ($^{11}\text{Li}$ [Iek93], $^{11}\text{Be}$ [Nak94] and $^{8}\text{B}$ [Mot94]), the angle $\varphi_{p\,-^{16}\,O}$ between the two fragments could be at maximum $\sim 30^\circ$ and one EXODET telescope should subtend a solid angle sufficient to detect both particles. On the other side, the relative angle between $^{16}$O and p should be large enough that the two particles hit different strips of the $\Delta E$ layer and are therefore separately detected. In any case, all the relationships between the geometry of the EXODET array and breakup reaction kinematics will be discussed in the following paragraph by the help of a Monte-Carlo simulation.
3.6.2 Breakup reactions measured with the EXODET apparatus

A Monte-Carlo simulation of the breakup process $^{17}\text{F} \rightarrow ^{16}\text{O} + p$ has been performed to investigate the capabilities of the powerful pixel-structure of the EXODET telescopes.

In particular, the most important aims of the simulation were:

1. to evaluate the limits of studying the breakup process $^{17}\text{F} \rightarrow ^{16}\text{O} + p$ at energy below the Coulomb barrier using only one EXODET telescope;

2. to estimate the geometrical efficiency of the EXODET pixel-structure when both breakup fragments $^{16}\text{O}$ and $p$ are detected;

3. to determine the dependence from the $^{17}\text{F}$ excitation energy of the previously calculated quantities.

Breakup process description

In our description of the breakup process, we first considered the $^{17}\text{F}$ scattering from a $^{208}\text{Pb}$ along a Rutherford trajectory and then its breakup into two particles, $^{16}\text{O} + p$, pulled apart by the Coulomb repulsion. Since the process takes place at energy below the Coulomb barrier, the nuclear interaction neither between the target and projectile nor subsequently between the target and the fragments were considered. We also assumed that the breakup occurs quite far (tens of fm) from the target, where the effects of the target Coulomb field on the outcoming fragments could be neglected (i.e. no post-acceleration was considered).

Two reference frames have been used to describe the whole interaction: the first (LAB) with origin at the target position and axes labeled $x$, $y$ and $z$ and the other (CM) with origin at the $^{17}\text{F}$ position and axes labeled $x'$, $y'$ and $z'$. The CM axes were orientated parallel to LAB ones. Since the distance between the two origins is a few tens of fm and our detectors are placed $\sim 2.5$ cm far from the target, the two origins could be assumed to be coincident for our purposes.

The reaction mechanism locates two bi-dimensional planes: one for the scattering process and the other for the breakup. The scattering plane is defined by the $^{17}\text{F}$ velocity vector before and after the scattering from the $^{208}\text{Pb}$ target. The incoming velocity is along the beam direction (axis $z$ of the LAB reference frame), while the outcoming vector $\vec{v}_F$ depends on the interaction with the target. Under this assumption, the scattering polar angle $\theta$ is defined as the angle between the vector $\vec{v}_F$ and the $z$ axis and the
scattering azimuthal angle $\phi$ is the angle between the scattering plane and the $(x-z)$ plane. The breakup plane is determined by the $^{16}$O velocity vector $\vec{v}_O'$ in the CM frame and the $z'$ axis. In this scheme the breakup polar angle $\theta'$ is the angle between the $\vec{v}_O'$ and the $z'$ axis and the breakup azimuthal angle $\phi'$ is the angle between the breakup plane and the $(x'-z')$ plane. Fig. 3.12 clearly illustrates the two reference frames together with the scattering and breakup angles.

![Reference systems](image)

Figure 3.12: Reference systems used for the description of the $^{17}$F scattering and its subsequent breakup. We indicated with the labels $v$, $\theta$ and $\phi$ the velocity, the polar angle and, respectively, the azimuthal angle in the LAB frame of the $^{17}$F ion scattered from a $^{208}$Pb target. Labels $v'$, $\theta'$ and $\phi'$ represent the velocity, the polar angle and, respectively, the azimuthal angle in the CM frame of one of the two fragments emitted in a $^{17}$F breakup process.

Using this formalism, we can write the components of the velocity vector $\vec{v}_F$ in the LAB frame:

$$
\begin{align*}
V_F & = \begin{cases} 
V_{x,F} = V_F \sin \theta \cos \phi \\
V_{y,F} = V_F \sin \theta \sin \phi \\
V_{z,F} = V_F \cos \theta
\end{cases}
\end{align*}
$$

and those of the vectors $\vec{v}_O'$ and $\vec{v}_p'$ in the CM system:

$$
\begin{align*}
V'_O & = \begin{cases} 
v'_{x,O} = v'_O \sin \theta' \cos \phi' \\
v'_{y,O} = v'_O \sin \theta' \sin \phi' \\
v'_{z,O} = v'_O \cos \theta'
\end{cases}
\end{align*}
$$

$$
\begin{align*}
V'_p & = \begin{cases} 
v'_{x,p} = -v'_p \sin \theta' \cos \phi' \\
v'_{y,p} = -v'_p \sin \theta' \sin \phi' \\
v'_{z,p} = -v'_p \cos \theta'
\end{cases}
\end{align*}
$$
In the last formulas, we assumed for the $p$ fragment an opposite direction with respect to that of the $^{16}\text{O}$, since in the CM frame the two fragments are emitted back-to-back because of momentum conservation.

Once we know the components of the $^{17}\text{F}$ velocity in the LAB system and those of the fragments in the CM system, we can calculate the components as well as the square module of the $^{16}\text{O}$ velocity in the LAB frame:

$$
V_O \left\{ \begin{array}{l}
V_{x,O} = V_{x,F} + v'_{x,O} = V_F \sin \theta \cos \phi + v'_O \sin \theta' \cos \phi' \\
V_{y,O} = V_{y,F} + v'_{y,O} = V_F \sin \theta \sin \phi + v'_O \sin \theta' \sin \phi' \\
V_{z,O} = V_{z,F} + v'_{z,O} = V_F \cos \theta + v'_O \cos \theta'
\end{array} \right.

\left( |V_O|^2 = \frac{2E}{M_p} \left( 1 + \frac{m_p}{m_O} \frac{E_{real}}{E} \right) + 2 \sqrt{\frac{m_p}{m_O} \frac{E_{real}}{E}} \left( \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\phi - \phi') \right) \right)
$$

and, similarly, for the proton:

$$
V_p \left\{ \begin{array}{l}
V_{x,p} = V_{x,F} + v'_{x,p} = V_F \sin \theta \cos \phi - v'_p \sin \theta' \cos \phi' \\
V_{y,p} = V_{y,F} + v'_{y,p} = V_F \sin \theta \sin \phi - v'_p \sin \theta' \sin \phi' \\
V_{z,p} = V_{z,F} + v'_{z,p} = V_F \cos \theta - v'_p \cos \theta'
\end{array} \right.

\left( |V_p|^2 = \frac{2E}{M_p} \left( 1 - \frac{m_p}{m_O} \frac{E_{real}}{E} \right) + 2 \sqrt{\frac{m_p}{m_O} \frac{E_{real}}{E}} \left( \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\phi - \phi') \right) \right)
$$

In this way we can now reconstruct the polar ($\theta_O$ and $\theta_p$) and the azimuthal ($\phi_O$ and $\phi_p$) angles of the breakup fragments in the LAB frame:

$$
\theta_O = \arccos \left( \frac{V_{z,O}}{|V_O|} \right) = \arccos \left( \frac{\sqrt{\frac{2E}{M_p} \left( \cos \theta + \sqrt{\frac{m_p}{m_O} \frac{E_{real}}{E}} \cos \theta' \right)}}{|V_O|} \right)
$$

(3.11)

$$
\theta_p = \arccos \left( \frac{V_{z,p}}{|V_p|} \right) = \arccos \left( \frac{\sqrt{\frac{2E}{M_p} \left( \cos \theta - \sqrt{\frac{m_p}{m_O} \frac{E_{real}}{E}} \cos \theta' \right)}}{|V_p|} \right)
$$

(3.12)

$$
\phi_O = \arctan \left( \frac{V_{y,O}}{V_{x,O}} \right) = \arctan \left( \frac{\sin \theta \sin \phi + \sqrt{\frac{m_p}{m_O} \frac{E_{real}}{E}} \sin \theta' \sin \phi'}{\sin \theta \cos \phi + \sqrt{\frac{m_p}{m_O} \frac{E_{real}}{E}} \sin \theta' \cos \phi'} \right)
$$

(3.13)

$$
\phi_p = \arctan \left( \frac{V_{y,p}}{V_{x,p}} \right) = \arctan \left( \frac{\sin \theta \sin \phi - \sqrt{\frac{m_p}{m_O} \frac{E_{real}}{E}} \sin \theta' \sin \phi'}{\sin \theta \cos \phi - \sqrt{\frac{m_p}{m_O} \frac{E_{real}}{E}} \sin \theta' \cos \phi'} \right)
$$

(3.14)

Combining the polar and azimuthal angles of the breakup fragments with the EXODET geometry, we can calculate the detector coordinates hit by each particle and determine the geometrical efficiency of the whole array for breakup measurements.
3.6. Breakup event analysis

Geometrical efficiency evaluation

In our simulation two completely random vector-directions were chosen. The first direction, with polar angle $\theta$ ranging from 0 to $\pi$ and azimuthal angle $\phi$ between 0 and $2\pi$, was used to describe the $^{17}$F nuclei undergoing breakup in the LAB frame. Since no information on the angular distribution of $^{17}$F breakup events was available and the limited statistics of our experiment did not allow to measure it, we assumed a uniform $\theta$ distribution. This was done in analogy with the system $^6$Li + $^{208}$Pb [Sig03], where the angular distributions of the $\alpha + d$ coincidence events were found to be practically flat over all the measured energy range around the Coulomb barrier.

The second random vector-direction, identified through the polar angle $\theta'$ and the azimuthal angle $\phi'$ within the same limits as for $\theta$ and $\phi$, was needed to describe the emission angle of the $^{16}$O (or $p$, since the two fragments are ejected at opposite directions) in the projectile CM frame. This assumption is realistic since in this reference system no privileged emission angle should exist.

We also remind the other assumptions made to simulate the whole process: no post acceleration of the breakup fragments was considered and no nuclear interaction either between the projectile and the target or between the target and the outgoing fragments was included. In addition, considering the kinematics of the ejected particles (see Table 3.2) and their energy loss in the $\Delta E$ stage (see Table 2.1), the $^{16}$O ion was always stopped in the first detector layer while the proton could reach the $E$ detector. For these reasons, in our scheme a coincidence event corresponded to a $\Delta E$ strip hit by the Oxygen ion and a $\Delta E - E$ pixel hit by the proton.

Fig. 3.13 shows the solid angle covered by a single EXODET telescope for the detection of the $^{17}$F breakup events as a function of the projectile excitation energy. This information is particularly useful for the evaluation of the solid angle coverage to be used in the normalization of the breakup cross section. The value reported at $E_{rel} = 0.0$ MeV refers to the elastic scattering process ($E_x = 0.0$ MeV). In this case the two fragments of the “unbroken” $^{17}$F ion obviously hit the same point of the telescope. The calculated geometrical efficiency is that previously estimated (see Secs. 2.3 and 2.4) for the front sides of telescope detectors. As the $^{17}$F excitation energy increases, the fragment kinematic cones start opening and the two ejected particles hit different points of the telescope. One of them could hit the inter-strip dead layers or simply “escape” from the detectors. These phenomena reduce the geometrical efficiency of the telescope and in Fig. 3.13 we can appreciate a quite strong dependence on the excitation energy, even for small $E_{rel}$ values. An accurate measurement of the $^{17}$F excitation energy distribution should
be needed in order to calculate the actual solid angle covered by the telescope, since the presence of possible sharp resonance could deeply modify an average evaluation.

An average value \( \Delta \Omega_{\text{breakup}} = (4.810 \pm 0.002) \text{ mb/sr} \), estimated over the energy range between 0.1 and 1.0 MeV above the breakup threshold where the largest contribution to the \(^{17}\text{F}\) excitation function is expected, has been used in the normalization of the breakup cross section.

The Monte-Carlo simulation was subsequently used to evaluate the ratio between coincidence events detected by a fixed EXODET telescope and all those detected in the whole array. Fig. 3.14 shows the percentage of this ratio as a function of the \(^{17}\text{F}\) excitation energy above the breakup threshold. As one can see the percentage is larger than 90\% for excitation energy up to \( \sim 1.3 \text{ MeV} \). In particular, in the energy range up to 1 MeV, where, similarly to other light RIB’s, the main contribution to the excitation energy distribution is expected, this ratio is even larger than 94\%. Of course, when the excitation energy gets larger the efficiency decreases, since the openings of kinematic cones are bigger and one of the two particles has a higher probability to hit a different telescope. However, we can conclude that using only one EXODET telescope we were able to detect almost all the available coincidences between an \(^{16}\text{O}\) ion and a proton for low \(^{17}\text{F}\) excitation energies.

Fig. 3.15 illustrates some ancillary information about the \( \Delta E \) strip ef-
Figure 3.14: Percentage of breakup coincidence events $^{16}O + p$ detected by a fixed EXODET telescope with respect to all coincidence events detected by the whole EXODET array as a function of the $^{17}F$ excitation energy above the breakup threshold $E_{rel}$. See text for more details about the point at $E_{rel} = 0.0$.

...efficiency for the detection of the $^{17}F$ breakup events. Border effects can differently affect the efficiency of each strip. In fact, for instance, if a $^{16}O$ ion hits a position close to the detector edges, at least half of the corresponding proton kinematic cone is not covered by the detector.

The upper panel of Fig. 3.15 shows the solid angle covered by each $\Delta E$ strip of a single EXODET telescope for the detection of the $^{17}F$ breakup events. Black points have been evaluated by considering only the strip-structure of the $\Delta E$ detector and neglecting the rear $E$ stage. The effects of the rear $E$ layer are taken into account by the red points. As we can see from the lower panel, where the percentage between these efficiencies has been plotted, the $E$ detector strip-structure reduces the geometrical efficiency of the first layer of about 20%, except for the last strips. In fact, since the two stages are placed at the same distance from the target support, if the beam spot is very small, scattered nuclei with high tilted trajectory can only be detected by the first stage.

As already observed, Fig. 3.15 also shows that the probability to detect both fragments decreases as the $^{17}F$ excitation energy increases. The lower panel shows that up to around 0.5 MeV above the breakup threshold, we can identify a plateau of $\sim 50$ strips, whose geometrical efficiency does not depend too much on the excitation energy. These strips correspond to the central...
Figure 3.15: Upper panel: solid angle covered by the $\Delta E$ strip of a single EXODET telescope for the detection of the $^{17}\text{F}$ breakup events as a function of $E_{\text{rel}}$. Black points refer to the $\Delta E$ strip efficiency without considering the rear $E$ detector. Red points describe the efficiency for the detection of elastic scattering events releasing energy in both stages. The other curves have been obtained for different $E_{\text{rel}}$. Lower panel: percentage ratio between the different curve reported in the upper panel and the solid angle coverage of each $\Delta E$ strip (black points).
region of the detector, where the only origin of the efficiency reduction arises from the strips edges. For the first strips, we clearly see that the efficiency rapidly falls down to around half of the elastic scattering one (red points), since half of the proton kinematic cone is rapidly moving outside the range covered by the telescope. At the opposite side, the decrease is smoother but starts much earlier, in fact the changes can be clearly seen for the last 30 strips. This is due to the combined effects of the coincidence events lost due to the detector edges (as for the first strips) and of the distance between the two stages, that hinders the detection of particles with high deflected trajectories.

As the $^{17}$F excitation energy gets larger, the width of the central plateau gets narrow. In fact this region of constant efficiency is no more visible starting from 2 MeV above the breakup threshold. At very high excitation energy (namely above 5 MeV), the efficiency is practically flat over all the strips. In this case the openings of the kinematic cones are so large, that the coincidence events have to be searched among all the detectors and the evaluation of the breakup cross section using only one EXODET telescope cannot be undertaken anymore.

Since in our experiment all the breakup events have been detected in the range between the strips no. 9 and no. 74 and, in analogy with the system $^6$Li + $^{208}$Pb [Maz03], the largest contribution is expected from $E_{rel}$-values lower than 1.0 MeV, no additional corrections to take into account the different geometrical efficiencies of the strips have been done.

We finally evaluated the possibility to detected both breakup fragments in the same strip of a $\Delta E$ detector. In this case, the digital information provided by the ASIC chip did not allow to distinguish the two particles, since only one $\Delta E$ strip has been hit and consequently there is only one ToT-value. Fig. 3.16 illustrates that this probability is higher than 5% only if $E_{rel}$ is lower than 0.2 MeV. Then the probability reaches an asymptotic value of 1.4-1.5% at around 3.0 excitation energy. However, the breakup event rate of our experiment was so low that we can a priori neglect the possibility of two fragments hitting the same $\Delta E$ strip.

### 3.6.3 Data analysis

At the light of the discussion done in the previous paragraphs and in Sec. 3.4, our detector array was found to be suited to perform an exclusive measurement of the breakup process $^{17}$F $\rightarrow ^{16}$O + p ($S_p = 0.601$ MeV).

Breakup events have been identified imposing the following conditions:

- two strips of the $\Delta E$ and only one of the $E$ detector layer should be
Figure 3.16: Detection probability of two $^{17}$F breakup fragments in the same strip of a $\Delta E$ detector as a function of $E_{rel}$.

hit;

- the JT of all the hit strips should be in the correlation peak ($JT \sim 10$);
- the ToT of one strip of the $\Delta E$ should be in the Oxygen range ($ToT = 8$), while the ToT of the other should be in the light particle range ($ToT = 2-4$);
- the total energy released in the $\Delta E$ detector should be comparable to that of the $^{17}$F and $^{17}$O elastic peaks (total energy of $^{16}$O added to the proton energy loss).

As we can see this is the case where the capabilities of the EXODET array together with the use of the ASIC chip are particularly remarkable.

The breakup differential cross section has been evaluated for an average polar angle $\bar{\theta}$ by normalizing the obtained yield to the Rutherford cross section with the following formula:

$$\frac{d\sigma}{d\Omega}(\bar{\theta}) = \frac{N_{breakup}}{\Delta\Omega_{breakup}} \sum_{\theta_F} \left( \frac{d\sigma}{d\Omega}(\theta_F) \right) \frac{\Delta\Omega_{\theta_F}}{N_F}$$

(3.15)

where $N_{breakup}$ is the total breakup event number and $\Delta\Omega_{breakup}$ is the solid angle coverage of a single EXODET telescope for the detection of the $^{17}$F breakup events with $E_{rel} \leq 1.0$ MeV, as discussed in the previous paragraph.
The second ratio is the normalization factor, as already explained for the formula 3.1.

As a result an average value of \((2.6 \pm 1.2) \text{ mb/sr}\) was estimated for the differential cross section of the breakup process \(^{17}\text{F} \rightarrow ^{16}\text{O} + p\) in the laboratory frame over the solid angle covered by the telescope in the backward direction \((\bar{\theta} \sim 125^\circ)\). The quoted error takes into account both the statistical error and the uncertainties in the evaluation of the geometrical efficiency via Monte-Carlo simulation (see Formula 2.1).
Chapter 4

Experiment II: $^{11}\text{Be} + ^{209}\text{Bi}$

4.1 $^{11}\text{Be}$ beam production at RIKEN

The $^{11}\text{Be}$ ($T_{1/2} = 13.8$ s) radioactive beam for this experiment was produced with the RIKEN Projectile-Fragment Separator (RIPS) in the RIKEN Accelerator Research Facility (RARF) located at Wako-shi in the Saitama Prefecture (Japan) close to Tokyo.

![Diagram of RIKEN Accelerator Research Facility (RARF)](image)

Figure 4.1: Layout of the RIKEN Accelerator Research Facility (RARF). The RIPS is installed across the two adjacent rooms D and E6.
The RIPS facility [Kub92] is essentially a doubly achromatic spectrometer, in angle and energy, with an intermediate dispersive focus between two dipoles that bend in the same direction. The system is installed across the two adjacent rooms D and E6 (see Fig. 4.1) at a fixed angle of 0° with respect to the RARF beam transport line. The whole facility consists in a series of optical elements (quadrupole, sextupole and dipole magnets) and detector arrays, schematically shown in Fig. 4.2.

Figure 4.2: Layout of the radioactive beam line RIPS. The dipoles are denoted by $D_1$ and $D_2$, the quadrupoles by $Q_1$-$Q_12$, the sextupoles $S_{X1}$-$S_{X4}$ and the focuses by $F_1$-$F_3$. As described in the text, the whole facility can be divided in three parts: the first between the production target chamber and the first focus $F_1$, the second one between $F_1$ and $F_2$ and the last one between $F_2$ and $F_3$.

### 4.1.1 $^{11}$Be secondary beam production

A primary beam, delivered by the RIKEN ring cyclotron with energy up to 135 MeV/nucleon and with intensity as high as $10^{12}$ particles/s, is focused on a thick production target mounted on a water-cooled holder. The radioactive secondary beam is produced via (In-Flight) projectile fragmentation. The fragments are collected and isotopically separated by the RIPS spectrometer and focused downstream, where a secondary target can be placed for RIB's
experiments. The solid angle acceptance of the spectrometer is 5 msr and the maximum momentum acceptance is 6%.

A very large variety of radioactive nuclei, even far from the valley of stability, can be produced with this technique. In addition, using inverse kinematic reactions, the projectile fragments are all emitted in small forward cones around the direction of the primary beam velocity. This kinematic feature together with the wide angular acceptance of RIPS and the availability of thick production targets, which obviously increases the yield of the fragmentation process, are the most important advantages of this secondary beam production method.

4.1.2 RIPS beam optics

As shown in Fig. 4.2, the RIPS beam line consists of two 45° dipoles (D1 and D2), twelve quadrupoles (Q1-Q12) and four sextupoles (SX1-SX4) arranged in a configuration of Q-Q-Q-SX-D-SX-Q-Q-SX-D-SX-Q-Q-Q-Q with three focuses at F1, F2 and F3. The whole facility can be divided into three independent sections by the intermediate focuses.

RIPS first section

The first section with a configuration of Q-Q-Q-SX-D-SX-Q ends with a dispersive focus at F1. The bending magnet D1 provides a magnetic analysis of the projectile fragments. Isotopes are selected according to their magnetic rigidity \( B_{\rho_D1} \), which is proportional to the \( A/Z \) ratio [Duf86], since all the fragmentation products have the same velocity:

\[
B_{\rho_D1} = 0.1439 \frac{A}{Z} \sqrt{E} \left( 1 + \frac{E}{2m_u} \right)^{1/2}
\]

(4.1)

where \( B_{\rho_D1} \) is expressed in Tm and \( E \) in MeV per nucleon, \( m_u \) being the atomic mass unit in MeV. The maximum field of the dipole is 1.6 T and the central ray \( \rho_{D1} \) radius is 3.6 m. This corresponds to maximum magnetic rigidity of 5.76 Tm, which is around 65% larger than that of the RIKEN ring cyclotron. Such a large bending power allows to use the highest possible primary beam energy even for fragments with large magnetic rigidity. This is necessary to enhance the yield of nuclei close to the neutron drip-line without losing the advantages of a thick target production and of the optimum kinematic focusing. The high energy primary beam can be stopped either in an appropriate Aluminum chamber inside the dipole or in a movable beam stopper at the end of D1. In order to reduce the background, the beam
stoppers are isolated from all the experimental downstream equipments by thick walls.

**RIPS second section**

The second section with a configuration Q-Q-Q-SX-D-SX-Q connects the focal planes F1 and F2. The ions selected by the first dipole undergo a thick Aluminium degrader plate and lose momentum differentially according to their \( A \) and \( Z \). The degrader (usually called achromatic degrader) has a wedge-shape properly designed to hold the achromaticity at the F2 location. The degrader decreases of about 30-40% the deflected fragments kinetic energy.

The second dipole D2 provides a second separation inside the subset of nuclei having \( A/Z \sim \text{constant} \). Since the momentum loss depends on the atomic number \( Z \) and on the fragment velocity, one can see that the second selection is roughly proportional to the ratio \( A^{2.5}/Z^{2.5} \) [Duf86]. This section compensates the dispersion of the first one providing a doubly achromatic focus, in angle and energy, at F2.

The quadrupoles placed in first two sections of the beam line are used to control the secondary beam dispersion and focusing. In particular, the quadrupole Q4 influences the dispersion at the focus F1, while the quadrupoles Q5 and Q6 are needed to reach the double achromaticity at the exit of the dipole D2. The two quadrupole triplets at the beginning of the RIPS beam line (Q1-Q2-Q3) and just in front of the F2 focal plane determine the optical conditions at F1 and F2, respectively. Finally, the sextupoles placed at the entrance and exit of both dipoles correct second-order aberrations.

**RIPS third section**

The third section with a quadrupole triplets provides the additional achromatic focal plane F3, where the experimental devices, such as secondary target or detectors, can be placed to perform nuclear reactions involving RIB’s. The main goal of the last section is to control the final focus of the secondary beam independently from the isotope separation provided by the previous sections.

Finally, when a low-energy beam is required, a second energy degrader (consisting of a rotatable absorber) can be placed at F2, and between F2 and F3 (\( \sim 5 \) m) the ToF of the secondary beam particles can be measured enabling beam energy reconstruction event-by-event.
4.2 Experimental set-up

4.2.1 RIPS equipment for $^{11}$Be production

![Diagram of RIPS equipment](image)

Figure 4.3: Layout of the radioactive beam line RIPS with all the equipments needed to provide the energy degradation of the secondary beam (achromatic degrader and rotatable degrader), to reconstruct the beam profile at the target position (PPACs $x$, $y$ position sensitive) and to get the ToF and energy calibration (PPACs, plastic scintillator and SSD). All the slit devices and the secondary reaction chamber where the EXODET array was placed are also shown.

In this experiment, the $^{11}$Be radioactive beam was produced via fragmentation of a $^{13}$C$^{6+}$ primary beam at 101.23 MeV/nucleon and $\sim$ 3 eμA intensity. The production target was a 12-mm-thick Beryllium foil, corresponding to a thickness of 2.22 g/cm$^2$. The produced 66.12 MeV/nucleon secondary beam passed through a 6-mm-thick wedge-shape Aluminium degrader plate in the achromatic mid-plane focus F1 of the the RIPS spectrometer. The secondary beam energy at the F1 exit was 47.65 MeV/nucleon. A first set of
slits, located in front of the wedge degrader, separated the selected ions from other reaction products. Fig. 4.3 schematically illustrates the beam optics and all the devices (degraders, slits and detectors) placed along the beam line.

After passing the second RIPS section, the $^{11}$Be beam reached the achromatic focus F2, where a second set of slits was located. To decrease the high kinetic energy to the required range by the experiment, a 5-mm-thick rotatable Aluminimum degrader plate was placed at the focal plane F2. The outcoming $^{11}$Be beam had a continuous energy spectrum with a parabolic distribution and a maximum at around 43 MeV. The thicknesses of the production target and of the degrader plates were chosen in order to optimize the beam intensity and to adjust the beam energy in the selected interval. The final secondary beam intensity measured at the F2 position was $\sim (1.0 \times 10^6)$ particle/s, integrated over the energy range 38-50 MeV. Then the beam was transported via the quadrupole triplet Q10-Q12 along the third RIPS section and eventually focused in F3, around 5 m downstream, where a final $(35 \times 35)$ mm$^2$ square collimator was located. The EXODET array was placed in the secondary reaction chamber $\sim 60$ cm downstream F3.

### 4.2.2 Secondary beam tagging system

Since the secondary beam had large energy spread and poor emittance, it was necessary to provide an event-by-event tracking in order to measure the particle incoming energy to reconstruct the beam profile at the target position. To do this, two sets of position sensitive Parallel Plate Avalanche Counters PPACa and PPACb [Kum01] were located in F3 at a distance of 304 mm.

These detectors consist of three plates, as shown in Fig. 4.4. An anode plate is placed between two cathodes which are connected to delay-lines. The cathodes consist of 1.0-mm wide strip and their orientations are orthogonal to each other, allowing a position resolution of $(1.0 \times 1.0)$ mm$^2$. There are 100 strips in both $x$ and $y$ directions, providing a total sensitivity area of $\sim (100.0 \times 100.0)$ mm$^2$. The strips are connected to delay-lines of 0.8 ns/mm yielding a total delay-time of 80 ns for the whole line. Signals are collected from each end of the delay line both in $x$ and $y$ direction and from the anode for a total amount of five signals for each PPAC. Both cathodes and anode are made by evaporation of Aluminimum with 30 nm thickness on a 1.5 $\mu$m Mylar foil. The distance between the anode and each cathode is 4 mm and is filled with $\text{C}_3\text{F}_8$ gas at a pressure of $\sim 8.9$ Torr for a total thickness of $\sim 2.5$ mg/cm$^2$ Carbon equivalent for each PPAC. During the experiment the operating anode bias was 850-920 V. The detector windows are made by 12
\[\mu m\] Mylar foils with 30 \(nm\) of Al evaporated on the surface.

The two PPACs were used on their own to determine the position hit by each incident particle at target plane, 682.5 mm downstream the second PPAC. On the other side, the two PPACs were also used together with a 1-mm-thick plastic scintillator located at F2 to perform a beam energy reconstruction via Time-of-Flight (ToF) measurement. The flight-path between the F2 position, where a plastic scintillator was giving the trigger signal, and the PPACa (PPACb) was 4.768 m (5.072 m) long, corresponding to ToF of 184.0 ns (195.7 ns) for a \(^{11}\)Be energy of 43 MeV. The ToF energy calibration was performed inserting an energy calibrated Silicon Solid State Detector (SSD) just behind the PPACs, as shown in Fig. 4.3. The Si SSD was 500 \(\mu m\) thick and was used for calibration purposes only, being removed from the beam line during the experiment. As we will see later, the effective \(^{11}\)Be beam energy was calculated from the ToF measurement taking into account the energy lost in both PPACs.

### 4.2.3 Detection equipment

In this experiment all the EXODET array was used. A cube of four two-stage telescopes were located at backward angles and another one in the forward distance. The system was not as symmetric as in the previous configura-
tion since the target holder was mounted on the mechanical support of the backward detectors. The distance between the detector active area and the target plane was 2.7 mm at backward angles and 6.7 mm for the forward detectors. The other geometrical features are equal to those shown in Fig. 2.10. The thickness of the used detectors was: 40 μm (70 μm) for six (two) ΔE layers and 500 μm for all the E stages. The two 70-μm-thick detectors were the left e the lower face of the ΔE cube at forward angles. The target was 3.03-mg/cm²-thick 209Bi evaporated on a 1-μm-thick Mylar foil.

![Graph](image)

**Figure 4.5:** Solid angles covered by an ideal 4π-geometry detector, the ΔE strip-structure and the ΔE – E pixel-structure of EXODET for the set-up geometry and beam profile used at RIKEN. Calculations have been performed for a Δθ binning of 1°.

Also in this case we performed a Monte-Carlo simulation to evaluate the total solid angle covered by each telescope of the EXODET array and the polar angles θ coverage for the present set-up geometry. As we will see in the following (see Sec. 4.4) the 11Be secondary beam had a quite large spot at the target position. The beam profile had a FWHM of 2.67 cm for the horizontal direction x and 1.95 cm along the vertical axis y. Moreover the beam did hit the target plane in its x central position, since the beam was slightly bended (about 1°) in the horizontal plane and the beam distribution showed a maximum at a distance of about 0.8 cm from the center in the left direction (upstream view). All these effects were taken into account in the simulation. The total solid angle covered by all the ΔE detectors and by the whole EXODET ΔE – E pixel-structure resulted to be 7.8787 ± 0.0002
sr (61.1 % of 4π sr) and 5.7577 ± 0.0002 sr (45.8 %), respectively. Fig. 4.5 shows the range of the polar angles θ subtended by the whole array estimated with the Monte-Carlo simulation for the present set-up geometry and beam profile. In particular one can appreciate the asymmetry between forward and backward angles, due to the fact that the target was closer to the backward hemisphere.

During the experiment, the data acquisition system was triggered by logical “OR” of all the ΔE energy signals. The collected data were: the energy signals coming from the not-segmented sides of the ΔE and E detectors, and the position information processed by the ASIC chip: hit strip number, Jitter Time (JT) and Time-over-Threshold (ToT). A commercial 32-channel Time to Digital Converter (TDC SILCENA model 9418/6-T) was also inserted in the VME bus to collect the time signals from both sides of the plastic scintillator, those from the anodes and the cathodes of the two PPACs placed at F3 and the BEAM OK information (see Par. 4.3.4 for more details on the BEAM OK signal). The TDC operated using a “common start” signal provided by the same trigger as for the energy signal treatment. All the time signals were adequately delayed to generate a stop signal for the corresponding TDC channel after the start information had reached the module.

For the ToF calibration, the SSD detector was placed at 0° with respect to the RIPS beam line in the F3 position, just in front of EXODET apparatus. During these runs the trigger for the data acquisition channel was given only by the energy signal of the detector itself.

Also in this case, all the data analysis has been performed with the package VISIM [Var89] using Linux as the operating system. Appendix C.1 will present one of the subroutines used for this data analysis.

### 4.3 ¹¹Be beam energy reconstruction

The heavy degradation needed to decrease the secondary beam energy to the range required by our experiment introduces a fairly large energy spread. In order to provide an event-by-event energy reconstruction of the incoming particles, we performed a Time-of-Flight measurement between the Plastic Scintillator located at the focal plane F2 and the PPACs placed at F3. The ToF has to be preliminarily energy calibrated using either primary or secondary beams with very sharp energy spreads. For this purpose several beams have been opportunistically selected changing the magnetic fields of the second dipole. Unfortunately the low-energy ¹¹Be secondary beam has a quite large energy lost (~10-20%) inside the PPACs. Therefore the ¹¹Be incident energy cannot be directly reconstructed from the ToF, since the energy loss due to
the PPACs has to be properly taken into account. The whole ToF calibration procedure required three steps:

1. the **energy calibration** of the Si SSD placed behind the two PPACs and in front of the secondary reaction chamber;

2. the **time** and **energy calibration** of the ToF;

3. the **evaluation** of the $^{11}$Be secondary beam **energy lost** into the two PPACs by comparing event-by-event the energy signal collected by the Si SSD with the energy reconstructed via ToF measurement.

### 4.3.1 Si SSD energy calibration

Several beams and set-up configurations of the RIPS spectrometer were used for the energy calibration of the Si SSD placed at the focus F3. After a first calibration performed with a commercial pulser and an $^{211}$Am source ($E_\alpha = 5.486$ MeV), in order to collect a few points in the energy range required by our experiment, the intensity of the $^{13}$C primary beam was reduced to a suitable range to avoid any damage of Si detector. Moreover all the equipments necessary to the secondary beam production, namely the production target, the wedge degrader between the dipoles and the rotatable absorber, and all the detectors (PPACs and plastic scintillator) needed for the beam tagging were removed from the beam line. The magnetic rigidity of the second dipole was set 895.5 mT to select the primary beam energy. In this way, the $^{13}$C with 101.29 MeV/nucleon beam energy could pass unperturbed through D2 and impinge directly on the Si detector. An energy loss of 23.925 MeV was obtained for such a beam inside a 0.5-mm-thick Si detector using the code S\textsc{rim} [S\textsc{rim}].

The Beryllium production target was subsequently inserted along the beam line and many secondary beams were produced due to the projectile fragmentation. The magnetic field of the second dipole was set to 603.1 mT to select ions with magnetic rigidity equal to 2.17 Tm. A lot of secondary beams with ratio $A/Z = 2$, namely $^4$He, $^6$Li, $^{10}$B and $^{12}$C, were produced using this procedure. Because of the energy selection of the dipoles (see formula 4.1) all these beams had the same energy of 55.2 MeV/nucleon. Other nuclei with the same magnetic rigidity but different mass-over-charge ratio were selected by the RIPS spectrometer at the same time: $^7$Be at 71.5 MeV/nucleon, $^{11}$B at 45.9 MeV/nucleon and $^{13}$C at 47.3 MeV/nucleon. All these secondary beams loose in the SSD detector an energy between 4.3 MeV ($^4$He) and 43.9 MeV ($^{13}$C) and they are very useful to provide a precise energy calibration of this detector. Table 4.1 lists the energy lost into a 0.5-mm-thick Si detector
4.3. $^{11}$Be beam energy reconstruction

by all these beams evaluated with the code SRIM. Fig. 4.6 shows a typical spectrum collected during this calibration run.

![Energy spectrum collected by the Si SSD. The magnetic field of the RIPS spectrometer was set in order to select the secondary beams with a magnetic rigidity of 2.17 Tm.](image)

**Figure 4.6:** Energy spectrum collected by the Si SSD. The magnetic field of the RIPS spectrometer was set in order to select the secondary beams with a magnetic rigidity of 2.17 Tm.

<table>
<thead>
<tr>
<th>beam</th>
<th>$B\rho_2$ (Tm)</th>
<th>$E$ (MeV/u)</th>
<th>$\Delta E$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{13}$C</td>
<td>3.22</td>
<td>101.29</td>
<td>23.93</td>
</tr>
<tr>
<td>$^{10}$B</td>
<td>2.17</td>
<td>55.22</td>
<td>26.51</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>2.17</td>
<td>55.22</td>
<td>38.60</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>2.17</td>
<td>71.51</td>
<td>13.72</td>
</tr>
<tr>
<td>$^{11}$B</td>
<td>2.17</td>
<td>45.86</td>
<td>30.84</td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>2.17</td>
<td>47.25</td>
<td>43.86</td>
</tr>
<tr>
<td>$^{11}$Be</td>
<td>3.28</td>
<td>47.65</td>
<td>18.88</td>
</tr>
</tbody>
</table>

**Table 4.1:** Primary and secondary beams used for the energy calibration of the Si SSD detector. The columns report the $D2$ magnetic rigidity necessary for the isotope selection, the outgoing kinetic energy and the energy loss in a 0.5-mm-thick Si detector evaluated with SRIM.

An additional energy point (already indicated in the Table 4.1) was obtained by inserting the wedge degrader along the beam line and by setting the magnetic rigidity to select a $^{11}$Be secondary beam with an energy of 47.65
MeV/nucleon. This was just the beam that will be later energy degraded by the F2 rotatable Al plate to match the energy range required by our experiment. The energy lost by such a beam into the SSD was 18.9 MeV.

All the evaluated energy points and the corresponding centroids in the SSD spectrum were fitted using a linear regression procedure. The agreement between the measured and the calibrated energy values was always better than 1%, with deviations between 50 keV and at maximum 200 keV. The main goal of this procedure was to provide an optimal energy calibration in the region between 30 and 50 MeV. In this range, the overall energy accuracy of the Si SSD was assumed to be better than 1%.

### 4.3.2 ToF calibration

To calibrate the ToF between the positions F2 and F3, we used the same beams as in the Si SSD energy calibration. In this case all the detectors needed to perform a ToF measurement, namely the F2 plastic scintillator and the F3 PPACs were inserted along the beam line. The Si SSD was left in front of the secondary reaction chamber, since it provided a very useful energy selection to disentangle the ToF spectrum when multiple secondary beams were passing through the spectrometer at the same time, as shown by Figs. 4.7 and 4.8.

The time signal from the plastic scintillator was collected with two photodiodes placed at opposite sides of the detector. In the following analysis we will consider the average of these two signals as ToF start signal. The stop signals were gathered from the anodes of the PPACs. In order to calculate the actual beam energy between the F2 and F3 positions, the code SRIM [Srim] was used to estimate the energy loss $\Delta E_1$ in the plastic scintillator (PLS). Second column of Table 4.2 shows that $\Delta E_1$ is a few tens of MeV for all the considered beams. Table 4.2 also reports the resulting kinetic energy $E_{F2-F3}$ (MeV) along the flight path, the expected ToF up to the PPACa and the experimental ToFe-values measured for all the beams. One can see that the experimental values decrease as the ToF-values deduced from the kinematics increase. This is due to the different delay-lines added to the PLS and to the PPACs time signals. In fact, we remind that all the time signals had to be delayed in order to reach the TDC module after the start signal provided by an EXODET $\Delta E$ signal or, in the calibration runs, by a signal collected by the Si SSD detector. In this case, a larger delay has been probably added to both signals coming from the plastic scintillator. However this additional delay only inverted the linear correspondence between the expected and the measured ToF-values, without affecting the incident energy reconstruction. The same effect has also been observed for the ToF measurement performed
### 4.3. $^{11}$Be beam energy reconstruction

![Graph showing ToF spectrum with peaks for $^6$Li-$^9$B and $^{11}$B-$^7$C]

**Figure 4.7:** ToF spectrum between the plastic scintillator located at the position F2 and the PPACa (ToFa), placed at the focus F3, 4.768 m downstream. This spectrum was collected setting the magnetic field of the second RIPS dipole at 603.1 mT to select different secondary beams produced by $^{13}$C fragmentation.

with the PPACb.

In principle, since two PPACs were placed at the F3 position, two ToF measurements, ToFa and ToFb between the PLS and the PPACa and, respectively, the PPACb, could be performed. Table 4.3 shows that the largest difference between the two reconstructed incident energies among all the beams used for the calibration, was smaller than 0.8%. In order to avoid the problems related to the evaluation of the energy lost $\Delta E_2$ into the first PPAC (last column of Table 4.2), we decided to use only the ToFa-value in the following analysis. This was not a serious problem for these high-energy beams, since for all of them $\Delta E_2$ was at maximum 0.2%, while a larger uncertainty could arise for the $^{11}$Be low-energy secondary beam. In fact, as we will see in the next paragraph, the $^{11}$Be energy lost into the PPACa estimated with SRIM for a 40-MeV beam energy is around 3.89 MeV, corresponding to $\sim$ 10% of its kinetic energy.

Finally, considering the agreement (see Table 4.2) between the incident energies determined via ToF measurement and those evaluated with the code SRIM for the seven beams used for the ToF calibration, we can conclude that the overall accuracy of the reconstructed energy was better than 1.0%.
Figure 4.8: Matrix of the ToFa and the energy lost in the Si SSD. The energy released by different secondary beams on the Si detector provided a very useful tool to disentangle the ToFa spectrum shown in Fig. 4.7.

<table>
<thead>
<tr>
<th>beam</th>
<th>$\Delta E_1$ (MeV)</th>
<th>$E_{F2-F3}$ (MeV)</th>
<th>ToF (ns)</th>
<th>ToFa (ns)</th>
<th>$\Delta E_2$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{13}$C</td>
<td>26.95</td>
<td>1289.83</td>
<td>34.46</td>
<td>178.8</td>
<td>0.69</td>
</tr>
<tr>
<td>$^7$Be</td>
<td>15.55</td>
<td>485.02</td>
<td>41.28</td>
<td>172.8</td>
<td>0.40</td>
</tr>
<tr>
<td>$^6$Li</td>
<td>11.00</td>
<td>320.23</td>
<td>47.04</td>
<td>168.2</td>
<td>0.28</td>
</tr>
<tr>
<td>$^{10}$B</td>
<td>30.27</td>
<td>521.93</td>
<td>47.54</td>
<td>166.5</td>
<td>0.80</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>44.11</td>
<td>618.54</td>
<td>47.81</td>
<td>166.4</td>
<td>1.16</td>
</tr>
<tr>
<td>$^{11}$Be</td>
<td>21.60</td>
<td>502.55</td>
<td>50.83</td>
<td>163.7</td>
<td>0.57</td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>50.35</td>
<td>563.90</td>
<td>52.12</td>
<td>162.6</td>
<td>1.35</td>
</tr>
<tr>
<td>$^{11}$B</td>
<td>35.41</td>
<td>469.05</td>
<td>52.59</td>
<td>161.9</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 4.2: Energy characteristics of the beams used for the calibration of the ToF between the F2 plastic scintillator and the F3 PPACa. Second column shows the energy lost in the plastic scintillator (estimated with the code SRLM), while third column indicates the resulting kinetic energy along the flight path. Fourth column contains the ToF-values deduced from the kinematics and the fifth the experimental values. Last column reports the energy lost in PPACa evaluated with SRLM.

4.3.3 $^{11}$Be incident energy reconstruction

Once the Si SSD was energy calibrated and the correspondence between ToF and the incident energy along the F2 - F3 path was established, it was possible
Table 4.3: Comparison between the reconstructed incident energies via ToF measurements and the kinetic energy evaluated with SRIM for the seven beams used for the ToF calibration.

to determine the energy of the low-energy $^{11}$Be secondary beam. This beam was produced by heavy degradation of the 47.65 MeV/nucleon through the Aluminium rotatable absorber at the focus F2. Fig. 4.9 shows that the resulting beam had a large energy spread (from 35 to 50 MeV total energy) and an event-by-event kinematic reconstruction of the projectile incident energy via ToF measurement was necessary.

![Graph showing SSD energy spectrum](image)

Figure 4.9: Si SSD energy spectrum collected during the calibration run with the heavy degraded $^{11}$Be secondary beam.

The relationship between the $^{11}$Be beam energy at the exit of the sec-
ond PPAC and the ToF-value was performed using all the tagging detectors placed along the beam-line and placing the Si SSD in front of the EXODET apparatus. We used the code SRRM to evaluate the energy loss into the two PPACs. Table 4.4 shows that $^{11}$Be ions lose a quite large amount of their kinetic energy. In fact when the $^{11}$Be incoming energy is 40 MeV, the particle looses more than 20% of its energy and the reduction is about 10% even at 60-MeV beam energy.

<table>
<thead>
<tr>
<th>$E_{F2-F3}$ (MeV)</th>
<th>$\Delta E_{PPACa}$ (MeV)</th>
<th>$\Delta E_{PPACb}$ (MeV)</th>
<th>$E_{EXODET}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.00</td>
<td>3.88</td>
<td>4.20</td>
<td>31.92</td>
</tr>
<tr>
<td>45.00</td>
<td>3.53</td>
<td>3.77</td>
<td>37.70</td>
</tr>
<tr>
<td>50.00</td>
<td>3.23</td>
<td>3.41</td>
<td>43.36</td>
</tr>
<tr>
<td>55.00</td>
<td>2.98</td>
<td>3.12</td>
<td>48.90</td>
</tr>
<tr>
<td>60.00</td>
<td>2.76</td>
<td>2.88</td>
<td>54.36</td>
</tr>
</tbody>
</table>

Table 4.4: Energy lost by $^{11}$Be particles into the two PPACs as a function of the incoming energy. Last column reports the projectile energies $E_{EXODET}$ at the exit of the second PPAC, that is the energy at the entrance of the EXODET detection apparatus. The values have been estimated with SRRM.

This energy reduction could systematically affect the energy reconstruction provided by the PPACb, since the distance between the two PPACs (304 mm) was covered by the particle with a smaller velocity with respect to that one between the plastic scintillator and the PPACa. For this reason the $^{11}$Be incoming energy was evaluated using only the ToFa. The energy lost in both the PPACs was experimentally taken into account comparing the energy reconstructed from the ToFa and the energy signal collected by Si SSD detector. Fig. 4.10 shows the behavior of the Si SSD energy signal as a function of the incoming energy reconstructed from ToF measurement. Since the energy lost increases as the incident energy decreases, the relationship between these two quantities could not be fitted using a simple linear regression. However we found that a good description could be achieved with a quadratic fit of the curve shown in Fig. 4.10. The second order correction, even much smaller than the linear coefficient, was able to reproduce quite well the experimental deviations from the linearity.

The systematic errors introduced by this calibration procedure and also the straggling effects due to the PPACs have not been studied in detail. However a qualitative analysis of Fig. 4.10 shows that, for example, a ToF-reconstructed energy $E_{ToFa} = 45$ MeV corresponds to an average energy signal in the Si SSD of $E_{SSD} \sim 42$ MeV, with a FWHM of $\sim 1.5$ MeV.
Therefore, the accuracy of the $^{11}$Be energy reconstruction was assumed $\sim 4\%$.

4.3.4 Pile-up rejection

During the experiment the counting rates of the F2 plastic scintillator ($\sim 1.0$ MHz) and of the F3 PPACs ($\sim 300$ kHz and $100$ kHz for PPACa and PPACb, respectively) were much higher than that of the events detected by EXODET (typical rate 20-30 Hz). In order to assign to each $^{11}$Be scattered ion its relative incident energy reconstructed via ToF measurement, we have to reject the pile-up events, i.e. the possibility that a second particle could hit either the plastic scintillator or the PPACs during the time of flight of the $^{11}$Be ion between the F2 and F3 locations ($\sim 200$ ns). Pile-up events (both pre- and post-pileup) were rejected by suited veto circuits with a rejection capability of $\sim 200$ ns [Wat02]. The rejected events were about $30\%$, as expected from the F2 event rate, indeed the plastic scintillator counting rate after pile-up rejection was around $700$ kHz.

The rejection system also provided a logical signal, named BEAM OK signal, that was sent to one channel of the TDC module. Fig. 4.11 shows a typical BEAM OK spectrum collected during the experiment. The bump centered at $350$ ns is due to the good events, while the background originates from random events caused by the high counting rate, especially at F2.
Beam ok signal ensures a correct energy reconstruction via ToF measurement only for events inside the bump and the wide structure of this peak reflects the energy spread of the incoming beam. In the forthcoming analysis we will consider only the events with $270 \text{ ns} \leq \text{Beam ok} \leq 420 \text{ ns}$.

![Graph showing Beam ok spectrum](image)

Figure 4.11: Typical Beam ok spectrum collected during the experiment. In the following data analysis only the events inside the bump ($270 \text{ ns} \leq \text{Beam ok} \leq 420 \text{ ns}$) will be considered.

### 4.4 $^{11}\text{Be}$ target position reconstruction

The heavy degradation provided by the rotatable absorber also makes worse the position resolution of the $^{11}\text{Be}$ secondary beam. As described in Sec. 4.2, two $x - y$ position sensitive PPACs have been placed along the beam line to monitor the beam spot at the F3 position and to provide a valid event-by-event reconstruction of the target position hit by the incoming particles.

This Section will present the procedures used for the position calibration of the two PPACs and the for the determination of the beam spot at the target plane.

#### 4.4.1 PPAC position calibration

As we saw in Sec. 4.2, each PPAC cathode had 100 1.0-mm wide strips connected to delay-lines and the time signals were collected from each end
of the delay line. Therefore the output data stream of each PPAC contained four “position” signals coming from the cathodes and an additional one from the unsegmented anode, obviously not used for the present analysis.

According to a reference frame with z axis along the beam direction, x axis in the horizontal plane with positive values on the left side (upstream view) and y axis in the vertical plane with positive values at the top, the position \((x_a, y_a)\) of the PPACa hit by the incoming particle could be reconstructed as follows:

\[
x_a = -K_{xa} \frac{T_{xla} - T_{x2a}}{2}
\]

\[
y_a = -K_{ya} \frac{T_{yla} - T_{y2a}}{2}
\]

where \(K_{xa}\) and \(K_{ya}\) are the slope factors for \(x\) and \(y\) direction, respectively. Since delay lines of \(~80\) ns/mm were used, typical values of the \(K\) factors are expected to be around 1.25 mm/ns. \(T_{xla}, T_{x2a}, T_{yla}\) and \(T_{y2a}\) corresponded to the delay times (in ns) of the so-called left, right, bottom and top signal, respectively. Similar formulas could be written also for the PPACb.

![Figure 4.12: \(x - y\) reconstructed matrix for the PPACa with the \(\alpha\)-source placed in front of the detector. The axes of the plot are still evaluated in ns. The edges of the PPACa are clearly visible.](image)

In order to determine the experimental value of the slope factors \(K\) for all the position information, we provided a geometrical calibration of both
PPACs. The faces of the PPACs were alternatively irradiated with an $^{241}$Am $\alpha$-source placed between the detectors. In this way it was possible to establish the positions corresponding to the detector edges. Figs. 4.12 and 4.13 show the $x - y$ matrices collected for the PPACa and the PPACb, respectively, during these calibration runs. In Fig. 4.13 one can clearly see the effects of the slits placed at the F3 position between the $\alpha$-source holder and the PPACb.

![Image of a matrix representing the $x_b - y_b$ reconstructed matrix with the $\alpha$-source placed in front of the PPACb. The axes are still quoted in ns. One can clearly see the detector edges and the corners due to the shadow of the F3 slits placed between the source holder and the PPACb itself.]

Assuming a covered area of $(100 \times 100)$ mm$^2$ for each PPAC, one could easily get the experimental values of the slope factors $K$, which are summarized in Table 4.5.

<table>
<thead>
<tr>
<th>detector</th>
<th>$K_x$ (mm/ns)</th>
<th>$K_y$ (mm/ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPACa</td>
<td>1.282</td>
<td>1.266</td>
</tr>
<tr>
<td>PPACb</td>
<td>1.266</td>
<td>1.282</td>
</tr>
</tbody>
</table>

Table 4.5: Experimental slope factors $K$ evaluated for the $x$ and $y$ delay-lines of both the PPACs.

All the values were systematically larger ($\sim 3\%$) than the expected ones.
The origin of these differences arose from a non-linear time-to-position dependence of the PPACs at the edge of the electrode [Yos04]. In fact, since the electric field is not uniform at the edge of the PPAC, a smaller time difference with the position change is measured at the electrode edges with respect to that obtained at the middle of the detector. For this reason a new set of $K$-values was determined for the middle parts of the electrodes via off-line measurements performed using masks with small holes (about 1 mm in diameter). The corresponding slope factor values were much closer to the expected value of $\sim 1.25 \text{ mm/ns}$ and are listed in Table 4.6.

<table>
<thead>
<tr>
<th>detector</th>
<th>$K_x$ (mm/ns)</th>
<th>$K_y$ (mm/ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPACa</td>
<td>1.246</td>
<td>1.254</td>
</tr>
<tr>
<td>PPACb</td>
<td>1.245</td>
<td>1.239</td>
</tr>
</tbody>
</table>

Table 4.6: Experimental slope factors $K$ evaluated for the $x$ and $y$ delay-lines of both the PPACs via off-line measurements performed using masks with small holes placed in the middle parts of the detectors.

Once the PPACs were position calibrated, one could evaluate the positions of the detector borders and the “electronic” center of each PPAC, i.e the PPAC point with the same delays from all the detector edges. These points did not correspond with the PPAC “geometrical” centers, i.e the points at the same distance from all the borders of the detector. For this reason an additional offset of a few mm was added to all the reconstructed positions $(x_a, y_a)$ and $(x_b, y_b)$ in order to make the two centers coincident. In addition, since the “geometrical” centers of both PPACs were not along the beam axis, a further offset was added to each position. In this way, the center of our reference frame was coincident with the optical axes used for the alignment of the EXODET apparatus.

### 4.4.2 Target position reconstruction

Since the distance between the two PPACs was 304 mm and the target was located 682.5 mm downstream the PPACb, the target position hit by each incoming particle could be reconstructed starting from the $x - y$ positions measured by the PPACs. Assuming a straight trajectory between the PPACs coordinates $(x_a, y_a)$ and $(x_b, y_b)$, the target position $(x_T, y_T)$ is given by the following formulas:

\[
x_T = x_b + \frac{682.5}{304} (x_b - x_a)
\] (4.4)
\[ y_T = y_b + \frac{682.5}{304} (y_b - y_0) \] \hspace{1cm} (4.5)

Table 4.7 summarizes the centroids and the FWHMs for the $^{13}$C primary beam and for the heavily degraded $^{11}$Be secondary beam along the $x$ and the $y$ axis, respectively. As we can see, both beams were very well centered at the PPACa position. Then the beams showed a shift of $\sim 2.5$ mm along the $x$ axis between the PPACs and this shift, obviously, got larger (7-8 mm) at the target position. We can conclude that both the primary and the secondary beam had an average deflection angle of $\sim 0.35^\circ$ along the $x$ axis.

On the other side, the two beams were practically aligned, within 2-3 mm, along the $y$ direction. In addition there seemed to be an additional focus between the PPACa and the target, since for both beams the centroids of the beam distribution at the target position were between the centroids at the PPACa and PPACb positions. The average deflection angle along this direction was event-by-event estimated to be $\sim 0.2^\circ$ for the primary beam and $\sim 0.1^\circ$ for the secondary beam.

<table>
<thead>
<tr>
<th>$x$ axis</th>
<th>$^{13}$C centroid</th>
<th>$^{13}$C FWHM</th>
<th>$^{11}$Be centroid</th>
<th>$^{11}$Be FWHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPACa</td>
<td>+0.2 mm</td>
<td>4.3 mm</td>
<td>+0.4 mm</td>
<td>19.8 mm</td>
</tr>
<tr>
<td>PPACb</td>
<td>+2.7 mm</td>
<td>3.9 mm</td>
<td>+3.1 mm</td>
<td>22.8 mm</td>
</tr>
<tr>
<td>Target</td>
<td>+7.5 mm</td>
<td>4.9 mm</td>
<td>+8.2 mm</td>
<td>26.7 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$y$ axis</th>
<th>$^{13}$C centroid</th>
<th>$^{13}$C FWHM</th>
<th>$^{11}$Be centroid</th>
<th>$^{11}$Be FWHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPACa</td>
<td>-0.4 mm</td>
<td>6.4 mm</td>
<td>-2.2 mm</td>
<td>33.3 mm</td>
</tr>
<tr>
<td>PPACb</td>
<td>-2.3 mm</td>
<td>5.4 mm</td>
<td>-3.6 mm</td>
<td>26.2 mm</td>
</tr>
<tr>
<td>Target</td>
<td>-1.8 mm</td>
<td>4.9 mm</td>
<td>-2.6 mm</td>
<td>19.5 mm</td>
</tr>
</tbody>
</table>

Table 4.7: Centroids and FWHMs along the $x$ (upper panel) and the $y$ (lower panel) axes for the $^{13}$C primary beam and for the degraded $^{11}$Be secondary beam at the PPAC positions and the reconstructed values at the target plane.

Table 4.7 also shows the effects of the heavy degradation of the $^{11}$Be secondary beam along the beam path. The beam spot at the target position of the $^{11}$Be beam was around $26.7 \text{ mm} (x) \times 19.5 \text{ mm} (y)$, where the primary beam diameter at the same location was around 5 mm. Fig. 4.14 shows the reconstructed distribution of the $^{13}$C primary beam at the target position, while the following figures illustrate the spot of the $^{11}$Be secondary beam measured with PPACa (Fig. 4.15), and PPACb (Fig. 4.16), and the reconstructed beam profile at the target position (Fig. 4.17). The effect of the
4.4. $^{11}$Be target position reconstruction

Figure 4.14: Reconstructed beam distribution at the target plane for the $^{13}$C primary beam. The beam diameter is $\sim 5$ mm.

Figure 4.15: Beam spot of the heavy degraded $^{11}$Be secondary beam measured with the PPACa. The rhomboidal shape is a consequence of the slits placed at the F3 position, since the trigger to data acquisition system was given by the Si SSD placed behind the slits.

slits placed along the beam line are clearly visible in the beam distributions
Figure 4.16: Beam spot of the heavy degraded $^{11}$Be secondary beam measured with the PPACb. The rhomboidal shape of the beam distribution is due to the slits placed at the F3 focus.

Figure 4.17: Reconstruction beam distribution at the target plane for the heavy degraded $^{11}$Be secondary beam.

at the PPAC positions.

Once the $^{11}$Be incident energy and target hit position have been event-
by-event reconstructed, it is possible to perform the analysis of the scattering process for the system $^{11}$Be + $^{209}$Bi.

4.5 Experimental results

![Energy spectra](image)

Figure 4.18: (a) $\Delta E$ and (b) $E$ energy spectra collected during the experiment with a forward telescope of the EXODET array.

Two typical energy spectra collected during the experiment with a forward telescope are shown in Fig. 4.18. In the upper panel we can see the $\Delta E$ spectrum in coincidence with the rear $E$ layer, while in the lower one there is the corresponding $E$ spectrum. Since the $\Delta E$ layers (see Table 2.1) were thin enough to let the scattered $^{11}$Be ions pass through, only the events that lost energy in both telescope detectors were considered in our analysis.

The $\Delta E$ spectrum contains a wide bump with a very large width (around 30 MeV), while the $E$ energy spectrum shows a decreasing trend as the energy lost gets higher. These broad structures originate from the $^{11}$Be scattering from a $^{209}$Bi target and the $\Delta E - E$ matrix (see Fig. 4.19) evidences a slight correlation between the energy losses by the particles into the two stages of the telescopes. The very wide nature of the bumps is strongly related to the large solid angle covered by the detector, the large energy spread of the incoming beam, the large beam spot at the target position and the energy
straggling due to the target thickness. The energy resolution of the EXODET detectors, estimated with an $^{241}$Am $\alpha$-source, was evaluated $\sim 4\%$ for the $\Delta E$ layers and $\sim 2\%$ for the $E$ stages.

Figure 4.19: $\Delta E - E$ matrix collected during the experiment with an EXODET telescope placed at forward angles.

Fig. 4.19 also evidences the presence of a high background in the energy spectra and, as we will see, the digital information processed by the ASIC chip are needed to efficiently suppress it. In the following analysis we will consider one telescope placed at forward angles. However, the same conclusions are valid also for all the other telescopes.

### 4.5.1 $\Delta E$ event analysis

The first digital information we considered was the Jitter Time of the $\Delta E$ ($JT_{\Delta E}$) signals. As we can see from Fig. 4.20, the $JT_{\Delta E}$ spectrum has a quite sharp peak around channel 10. This ensures the time correlation of most of the $\Delta E$ events. In order to avoid spurious and uncorrelated signals only the events with $9 \leq JT_{\Delta E} \leq 11$ were taken into account. Considering the 15 MHz-frequency of the ASIC chip clock, this selection means that correlated events have a Jitter Time between 600 ns ($9 \times 67$ ns) and 733 ns ($11 \times 67$ ns).

Fig. 4.21 shows the Time over Threshold spectrum of the $\Delta E$ ($ToT_{\Delta E}$) events before (upper panel) and after (lower panel) the selection of the $JT_{\Delta E}$
4.5. Experimental results

Figure 4.20: Jitter Time spectrum of the events collected with a $\Delta E$ EX-ODET detector placed at forward angles.

Figure 4.21: Time over Threshold spectra collected with a forward $\Delta E$ detector before (a) and after (b) the selection of the Jitter Time ($JT_{\Delta E}$) correlation peak. The original spectrum has a very sharp maximum at 467 ns ($7 \times 67$ ns) and smaller contributions coming from other channels.
After the $JT_{\Delta E}$ selection, a net reduction is clearly visible for ToT-values ranging from 2 to 6, while the events with ToT larger than 600 ns ($9 \times 67$ ns) are completely suppressed. Only the structure between the ToT-values 7 and 8 remains practically unmodified by the imposed gate. Following the experience of the $^{17}$F experiment (see Sec. 3.4), we can eventually conclude that these events are candidates to be “good” $^{11}$Be scattering events.

4.5.2 $E$ event analysis

![Time over Threshold spectrum](image)

Figure 4.22: *Time over Threshold spectrum for the forward $E$ detector obtained by imposing two gates on the $\Delta E$ digital information processes by the chip ASIC: $JT_{\Delta E} = 9-11$ and $ToT_{\Delta E} = 7-8$.*

Let us now consider the digital information processed by the $E$ detector. Fig. 4.22 shows the Time over Threshold spectrum of the $E$ events (ToT$_{ER}$) correlated with good $^{11}$Be $\Delta E$ events ($9 \leq JT_{\Delta E} \leq 11$ and $7 \leq ToT_{\Delta E} \leq 8$). The spectrum has two components: one with $2 \leq ToT_{ER} \leq 4$, which has the largest population, and a second one with $7 \leq ToT_{ER} \leq 8$. The nature of each part can be understood by looking at the Jitter Time ($JT_{ER}$) spectrum of these events.

Upper panel of Fig. 4.23 clearly shows that the Jitter Time spectrum of the $E$ events with $2 \leq ToT_{ER} \leq 4$ is rather flat and no correlation peak around 667 ns ($10 \times 67$ ns) appears. This means that these events have no time correlation with the trigger signal of the data acquisition system. They most likely originate either from $\beta$-decay of $^{11}$Be ions implanted into
the 500-μm-thick $E$ layer or from α-decay or α-evaporation after compound nucleus reactions. Fig. 4.24a shows the $E$ energy spectrum of these events. There is a quite sharp peak at very low energy ($E \sim 1 - 3$ MeV) then a long tail extended up to $\sim 40$-$50$ MeV. This spectrum is not very significant since only one energy signal is gathered from the continuous rear side of the detector, even if more than one strip has been simultaneously hit. In these cases the energy signal is the sum of the energy lost by all the hitting particles. However, since the chip separately analyzes all the strip signals, a small ToT-value ensures that the energy released in the considered strip is rather small. The background is higher for the $E$ detector rather than for the $\Delta E$ one, since its thickness is about one order of magnitude larger and the $^{11}$Be radioactive ions are fully stopped in the second layer.

On the other side, Fig. 4.23b shows that for $7 \leq \text{ToT}_E \leq 8$, the Jitter Time spectrum of the $E$ events is sharply peaked at around 667 ns ($10 \times 67$ ns). This confirms the time-correlation of all the $E$ signals with $7 \leq \text{ToT}_E \leq 8$ and $9 \leq \text{JT}_E \leq 11$. These two gates together with those imposed for the $\Delta E$ digital information provided a very useful background suppression.
Figure 4.24: Energy spectra collected by E layer of a forward telescope with different gates on the $T_0 T_{ER}$ signal: a) $T_0 T_{ER} = 2-4$, b) $T_0 T_{ER} = 7$ and c) $T_0 T_{ER} = 8$. The following selections on the other digital information provided by ASIC chip are the same for all the panels: $J T_{\Delta E} = 9-11$, $T_0 T_{\Delta E} = 7-8$ and $J T_{ER} = 9-11$.

and will be used in the following analysis to select the $^{11}$Be scattering events.

Fig. 4.24 shows the $E$ energy spectra for $T_0 T_{ER} = 7$ (middle panel) and $T_0 T_{ER} = 8$ (bottom panel). The bump corresponding to the $^{11}$Be scattering events is clearly seen in both spectra. In particular, the comparison between them shows that the events with $T_0 T_{ER} = 7$ have an average energy $\sim 15$ MeV, while those with $T_0 T_{ER} = 8$ have the average energy $\sim 25$ MeV. This is a clear indication of the ToT energy resolution in this energy range.

4.5.3 $\Delta E$ and $E$ hit strip number

Up to now we have not used yet the digital information on the number of simultaneously hit strips of the same detector. Since we are dealing with a scattering process, one would expect that for each collected event only a single strip has been hit. Unfortunately, many strips of the same detector usually recorded at the same time an energy signal above the chip internal threshold. In fact since the chip used was originally developed for particle physics experiment, a proper signal attenuator was needed to match the chip
required energy range. In order to reach the highest possible chip efficiency, these externally settable thresholds were kept so low that the strip signal could be sometimes triggered either by the noise or in case by a signal induced from a neighboring strip.

The number of strips that typically processed an energy signal over the chip threshold ranges from 1 to 3 for the \( \Delta E \) layer and from 4 to 7 for the \( E \) stage. However, we have already observed that the analysis of the Jitter Time and the Time over Threshold provides a very useful tool to distinguish between trigger correlated and uncorrelated events. In fact, if we select the events with both \( 9 \leq JT \leq 11 \) and \( 7 \leq ToT \leq 8 \), even if they originate from events where more than one strip has simultaneously processed an energy signal, \( \sim 97\% \) \((98.2\%\) of the \( \Delta E \) \((E)\) events have only one strip with all the chip parameters within the selected ranges. This confirms that a constrain on the number of hit strips would not improve our background suppression, while it might result in a severe suppression of good events.

### 4.6 \( ^{11}\text{Be} \) scattering event analysis

![Graph](image)

**Figure 4.25:** (a) \( \Delta E \) and (b) corresponding \( E \) energy spectra collected with a forward EXODET telescope after the selection of the digital information provided by the chip ASIC: \( 9 \leq JT \leq 11 \) and \( 7 \leq ToT \leq 8 \).
Figure 4.26: $\Delta E - E$ matrix gathered with an EXODET telescope located at forward angles after imposing the following gates on the Jitter Time and the Time over Threshold on the collected signals: $9 \leq JT \leq 11$ and $7 \leq ToT \leq 8$.

Fig. 4.25 shows the $\Delta E$ (upper panel) and the corresponding $E$ energy spectra (bottom panel) for a telescope placed at forward angles after the selection of the previously discussed JT and ToT gates. The $\Delta E$ spectrum has an asymmetric Gaussian shape centered at about 25 MeV and with a long tail at high energy. The $E$ spectrum also shows a nearly Gaussian distribution, slightly broader than the $\Delta E$ one, centered at about 15 MeV. The comparison with Fig. 4.18 shows that the digital information processed by the ASIC chip are very useful to suppress the background. This effect is particularly evident for the $E$ spectrum, where the large edge at low energy practically disappears and also the event distribution results to be deeply modified by the gates. The consequences of the background suppression are also very remarkable for the $\Delta E - E$ matrix, as we can see comparing Figs. 4.19 and 4.26. In spite of the large energy spread of the incoming particles and the large beam spot at the target position, we can appreciate that only the $^{11}$Be banana practically survives the selections. Even if the imposed gates drastically reduce the collected statistics, the digital analysis allows to select only the correlated events among a much larger amount (at least one order of magnitude, as we can deduce from the comparison of the $\Delta E$ and $E$ projections in Figs. 4.18 and 4.25) of spurious signals.
4.6.1 Data analysis

At the light of the results obtained in the previous section, the following requirements have been imposed on the digital information provided by the ASIC chip, for the $\Delta E - E$ correlated $^{11}$Be scattering events:

- the Jitter Time of both the $\Delta E$ and $E$ signals have to be in the time-correlation peak (9-11);

- the Time over Threshold of both the $\Delta E$ and $E$ signals have to range between 7 and 8.

No additional selections on the energy lost in the telescope stages are needed, as for the $^{17}$F scattering event analysis (see Sec. 3.5).

Our experimental energy resolution did not allow to distinguish between “pure” elastic scattering events from quasi elastic events (with either $^{11}$Be first excited state at 0.320 MeV or $^{209}$Bi first excited states at 0.896 MeV and 1.609 MeV being populated in the interaction). The contributions to the quasi-elastic cross section arising from these inelastic excitations were estimated via DWBA calculation (see Par. 5.1.3). Target excitations were found to be negligible in a first approximation in comparison with the pure elastic scattering differential cross section, whereas the contribution from the projectile excitation resulted to be up to 30% of the elastic scattering angular distribution.

Once we know the $\Delta E$ strip and the $E$ strip hit by the scattered $^{11}$Be, we can immediately determine the EXODET pixel and the detector position $(x_D, y_D, z_D)$ hit by the particle, within an uncertainty of $(0.5 \times 0.5)$ mm$^2$. According to a reference frame with the $z$ axis along the beam line and the target plane located at $z_T = 0$, the detector position together with the knowledge of the target position $(x_T, y_T, z_T = 0)$ hit by the incoming particle allow the reconstruction of the polar scattering angle $\theta$ in the laboratory frame, through the following formula:

$$\theta = \arctan \left( \frac{\sqrt{(x_D - x_T)^2 + (y_D - y_T)^2}}{z_D} \right) \quad (4.6)$$

We remind the accuracy of measurement of the target position, performed with the PPACs (see Sec. 4.4), is $(1.0 \times 1.0)$ mm$^2$. Considering the uncertainties of all the quantities needed for the $\theta$ evaluation, an overall accuracy of about 2° was estimated for the resulting scattering angle. Fig. 4.27 shows the $\theta$ reconstructed spectrum over all the beam energies. In spite of the large beam energy spread, the efficiency reduction due to not working strips (responsible for the holes at $\theta \sim 55°$ and 70° in the $\theta$ spectrum) and the
Figure 4.27: Reconstructed $\theta$ scattering angles distribution in the laboratory frame calculated for all $^{11}$Be beam energies.

different solid angle coverage of each angle; a trend similar to the Rutherford differential cross section could be identified. The hole at $\theta \sim 90^\circ$ is due to the target holder shadow.

Figure 4.28: Spectrum of the $^{11}$Be incoming beam energy.
In order to evaluate the $^{11}\text{Be}$ scattering angular distributions and to compare them with those measured for the system $^9\text{Be} + ^{209}\text{Bi}$, the continuum spectrum of the incoming energy (shown in Fig. 4.28) was subdivided into 2-MeV bins. This value was found to be a good compromise between statistics, energy resolution of the resulting angular distributions and accuracy of the incoming beam energy reconstruction via ToF measurements ($\sim 4\%$ at 42-MeV beam energy). The reduction of the effective beam energy interaction due to the target thickness was not taken into account, since it was rather small in comparison with both energy binning and incident energy uncertainty. In fact the energy loss for 42-MeV $^{11}\text{Be}$ ions into a 1.5-mg/cm$^2$-thick $^{209}\text{Bi}$ foil (half of the target thickness) results to be $\sim 0.8$ MeV.

![Figure 4.29](image)

Figure 4.29: Total energy spectrum (sum of the energy released in the $\Delta E$ and $E$ telescope stages) for the six angular bins used to evaluate the the scattering differential cross section for the system $^{11}\text{Be} + ^{209}\text{Bi}$ at 40-MeV beam energy.

The range of the polar angles $\theta$ has been divided into $10^\circ$-bins. Figs. 4.29 and 4.30 shows the $^{11}\text{Be}$ total energy spectrum (sum of the energy released in
Figure 4.30: *Total energy spectrum for the six angular bins used to evaluate the scattering angular distribution for the system $^{11}$Be + $^{209}$Bi at 42-MeV beam energy.*

the $\Delta E$ and $E$ telescope stages) for the six angular bins used to evaluate the scattering angular distributions at 40- and 42-MeV beam energy (the spectra relative to the other beam energies are reported in Appendix C.2). All the spectra have a quite Gaussian shapes, in particular those at forward angles, with very large widths ($\sim$ 7-10 MeV). The origins of these broad structures are manifold: the energy resolution of the incoming beam due to the 2-MeV binning ($\sim$ 5%), the accuracy in the reconstruction of the incident energy via ToF measurements ($\sim$ 4%), the energy resolutions of the detectors ($\sim$ 4% for the $\Delta E$ layers and $\sim$ 2% for the $E$ ones), the energy lost into the target ($\sim$ 2%) and finally the quite large angular range (10°) necessary to get enough statistic for each bin.

All these facts translate in a lower quality of resulting scattering angular distributions. In particular the values evaluated at backward angles for high beam energies, where even with such a large angular binning we were not
able to achieve a sufficient statistics (see Appendix C.2), suffer of both large statistics and systematical uncertainties. In addition, we have to underline that these are the energies and the angular ranges where the influence of non-elastic channels on the elastic scattering process is larger and therefore where the most rapid variations of scattering differential cross sections are expected.

### 4.6.2 Data normalization

Data have been normalized using the $^{11}$Be low energy events under the assumption that the measured cross section at 38-MeV beam energy was purely Rutherford. DWBA and Coupled-Channel calculations ensure the this hypothesis is realistic up to $\theta \sim 120^\circ / 130^\circ$, whereas it introduces a systematic error ($\sim +10-20\%$) for large scattering angles and the resulting angular distributions could be slightly overestimated.

Data normalization has been performed starting from the relationship between the counts per second $N(\bar{\theta}, E_0)$ at a certain angle $\bar{\theta}$ for a selected beam energy $E_0$ and the corresponding differential cross section $\frac{d\sigma}{d\Omega}(\bar{\theta}, E_0)$:

$$N(\bar{\theta}, E_0) = \frac{d\sigma}{d\Omega}(\bar{\theta}, E_0) \Delta\Omega(\bar{\theta}) \eta(\bar{\theta}, E_0) i(E_0) T$$

(4.7)

with $\Delta\Omega(\bar{\theta})$ solid angle covered by the polar angle $\bar{\theta}$, $\eta(\bar{\theta})$ detector efficiency at the angle $\bar{\theta}$, $i(E_0)$ beam intensity at $E_0$ and $T$ target thickness. If the differential cross section $\frac{d\sigma}{d\Omega}$ does not vary too much over a sufficiently small range of polar angles with average value $\bar{\theta}$ and assuming that the detector efficiency $\eta$ does not depend on the beam energy $E_0$, we can write:

$$\frac{N(\bar{\theta}, E_1)}{N(\bar{\theta}, E_0)} = \frac{\frac{d\sigma}{d\Omega}(\bar{\theta}, E_1) i(E_1)}{\frac{d\sigma}{d\Omega}(\bar{\theta}, E_0) i(E_0)}$$

(4.8)

Equation 4.8 can also be rewritten in this way:

$$\frac{d\sigma}{d\Omega}(\bar{\theta}, E_1) = N(\bar{\theta}, E_1) i(E_1) \frac{d\sigma}{d\Omega}(\bar{\theta}, E_0)$$

(4.9)

If we assume that the differential cross section at $E_0$ beam energy and at the polar angle $\bar{\theta}$ is purely Rutherford, i.e. $\frac{d\sigma}{d\Omega}(\bar{\theta}, E_0) = \frac{d\sigma_R}{d\Omega}(\bar{\theta}, E_0)$, Equation 4.9 becomes:

$$\frac{d\sigma}{d\Omega}(\bar{\theta}, E_1) = N(\bar{\theta}, E_1) i(E_1) \frac{d\sigma_R}{d\Omega}(\bar{\theta}, E_0)$$

(4.10)

Dividing both terms of Equation 4.10 by the Rutherford cross section at the polar angle $\bar{\theta}$ and at beam energy $E_1$, we can easily get:
\[
\frac{d\sigma}{d\sigma_R}(\bar{\theta}, E_1) = \frac{N(\bar{\theta}, E_1) i(E_0) d\sigma_R(\bar{\theta}, E_0)}{N(\theta, E_0) i(E_1) d\sigma_R(\theta, E_1)} \tag{4.11}
\]

\[
= \frac{N(\bar{\theta}, E_1) i(E_0)}{N(\theta, E_0) i(E_1)} \left(\frac{1}{E_1}\right)^2 \tag{4.12}
\]

\[
= \frac{N(\bar{\theta}, E_1) i(E_0)}{N(\theta, E_0) i(E_1)} \left(\frac{E_1}{E_0}\right)^2 \tag{4.13}
\]

In this way the differential cross section for the energy bin centered at \(E_1\) and \(\bar{\theta}\) can be easily determined from the ratio between the events of the energy bins \(E_1\) and \(E_0\) (in the present case \(E_0 = 38\) MeV) at \(\bar{\theta}\) and from the ratio between the beam intensities.

![Graph](image_url)

**Figure 4.31:** Angular distributions for the \(^{11}\text{Be}\) scattering process for a \(^{209}\text{Bi}\) target in the energy range between 40 and 48 MeV.

Fig. 4.31 and Table 4.8 show the resulting \(^{11}\text{Be}\) scattering differential cross sections for beam energies ranging from 40 to 48 MeV. Because of the limited statistics only four points have been deduced at backward angles. Due to some uncertainties in the evaluation of the detector efficiency in the region \(55^\circ \leq \theta \leq 85^\circ\) still under investigation, only two values in forward direction have been reported. The plotted errors take into account only the statistical errors for the \(y\) axis and one can see that at backward angles they
<table>
<thead>
<tr>
<th></th>
<th>$40\text{ MeV}$</th>
<th>$42\text{ MeV}$</th>
<th>$44\text{ MeV}$</th>
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<tr>
<td>$\theta_{\text{cm}}$</td>
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<td>$\theta_{\text{cm}}$</td>
<td>$\frac{d\sigma}{d\theta_R}$</td>
</tr>
<tr>
<td>39.7</td>
<td>0.98 ± 0.05</td>
<td>39.7</td>
<td>1.02 ± 0.05</td>
</tr>
<tr>
<td>48.8</td>
<td>1.02 ± 0.06</td>
<td>48.8</td>
<td>0.98 ± 0.05</td>
</tr>
<tr>
<td>100.4</td>
<td>0.82 ± 0.11</td>
<td>100.4</td>
<td>0.53 ± 0.08</td>
</tr>
<tr>
<td>110.2</td>
<td>0.77 ± 0.10</td>
<td>110.2</td>
<td>0.61 ± 0.08</td>
</tr>
<tr>
<td>119.9</td>
<td>0.78 ± 0.11</td>
<td>119.9</td>
<td>0.59 ± 0.09</td>
</tr>
<tr>
<td>129.6</td>
<td>0.87 ± 0.18</td>
<td>129.6</td>
<td>0.67 ± 0.13</td>
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<th>$48\text{ MeV}$</th>
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<td>$\frac{d\sigma}{d\theta_R}$</td>
<td>$\theta_{\text{cm}}$</td>
</tr>
<tr>
<td>39.7</td>
<td>1.00 ± 0.05</td>
<td>39.7</td>
</tr>
<tr>
<td>48.8</td>
<td>1.00 ± 0.05</td>
<td>48.8</td>
</tr>
<tr>
<td>100.4</td>
<td>0.32 ± 0.05</td>
<td>100.4</td>
</tr>
<tr>
<td>110.2</td>
<td>0.23 ± 0.04</td>
<td>110.2</td>
</tr>
<tr>
<td>119.9</td>
<td>0.19 ± 0.04</td>
<td>119.9</td>
</tr>
<tr>
<td>129.6</td>
<td>0.20 ± 0.06</td>
<td>129.6</td>
</tr>
</tbody>
</table>

Table 4.8: Angular distributions for the $^{11}\text{Be}$ scattering process from a $^{209}\text{Bi}$ target for beam energies from 40 MeV to 48 MeV. The quoted errors consider only the statistical uncertainties.

sometimes account for uncertainties larger than 30%. On the other side, since the polar angles have been grouped into $10^\circ$-bins, an average indetermination of $5^\circ$ was assumed for the plotted $\theta$ angles.

The angular distributions result to be rather flat at 40- and 42-MeV beam energy, whereas the influence of non-elastic channels is clearly visible at backward angles for higher incident energies. Rapid variations in the differential cross sections could not be observed due to the large angular range subtended by each bin and because of the uncertainties into the reconstruction and binning of the incident energy. Even if a better energy resolution of the secondary beam could improve the quality of the resulting angular distributions, present data are sufficient to a primary investigation of the effects of the $^{11}\text{Be}$ halo structure and low binding energy on the elastic scattering process.
Chapter 5

Theoretical Analysis

The experimental data for the scattering processes of $^{17}$F nuclei from a $^{208}$Pb target and $^{11}$Be ions from $^{239}$Bi were analyzed in the optical model framework. These two projectiles are quite interesting since they are very weakly bound ($S_p = 0.601$ MeV for $^{17}$F and $S_n = 0.504$ MeV for $^{11}$Be) and because of their halo structures. In fact, $^{11}$Be has an r.m.s. radius much larger [Tan85] than that foreseen by the $r_0A^{1/3}$ systematic with $r_0 \sim 1.2$ fm, while $^{17}$F has a well-established halo structure in its first excited state at 0.4953 MeV [Mor97].

In the energy range around the Coulomb barrier the analysis of complex ion elastic scattering usually leads to non-unique parameter sets for the potential, making impossible an accurate comparison of results obtained for different systems or for the same reaction at different beam energies. However, the potential values at the strong absorption radii and the reaction cross sections are usually well defined and rather independent from all the fitting ambiguities.

Therefore the main goals of the optical model analysis performed in this Chapter were:

1. to investigate the influence of the low binding energy and of the halo structure on the interaction potential;

2. to get consistent optical model parameter sets;

3. to define the potential around the strong absorption radius;

4. to determine the reaction cross section induced by loosely bound projectiles in the energy range around the Coulomb;

5. to compare these results with those obtained for similar mass systems, particularly with those involving stable well-bound isotopes.
5.1 Optical model analysis

5.1.1 $^{17}$F + $^{208}$Pb system at 90.4-MeV beam energy

The scattering data for the system $^{17}$F + $^{208}$Pb at 90.4-MeV beam energy have been fitted in the framework of the optical model using the specific subroutines of the code FRESCO [Tho88]. In order to provide an optimal comparison with the stable Fluorine isotope, we followed the same procedure adopted in [Lin01] for the analysis of the scattering process for the reaction $^{19}$F + $^{208}$Pb.

![Graph](image)

Figure 5.1: Experimental angular distribution for the $^{17}$F scattering process from a $^{208}$Pb target at 90.4-MeV beam energy and optical model best-fit obtained with $r_0 = 1.24$ fm and $a_0 = 0.53$ fm. The red curve was obtained adding the cross sections for the inelastic excitation to the first $^{17}$F excited state at 0.4953 MeV to that of the “pure” elastic scattering process. The blue curve includes also the excitation to the first $^{208}$Pb level at 2.614 MeV.

The nuclear interaction potential between the projectile and the target has been described with a Woods-Saxon well (see Formula 5.1) for both the real $V(r)$ and the imaginary part $W(r)$:

$$V(r) + iW(r) = - \left( \frac{V_0}{1 + e^{(r - r_{0w}/A^{1/3})/\alpha_w}} + i \frac{W_0}{1 + e^{(r - r_{0w}/A^{1/3})/\alpha_w}} \right)$$

(5.1)
The optical model best-fit analysis was performed fixing the radii \( r_{0w} = r_{0v} = r_0 \) and the diffusenesses \( a_v = a_w = a_0 \) and varying the potential depths \( V_0 \) and \( W_0 \).

In order to check the influence of each potential parameter, different fits were done. We selected two grids: one with fixed radius parameter \( r_0 = 1.20 \) and \( 1.24 \) fm) and the other with four fixed values for the diffuseness parameter \( a_0 = 0.43, 0.48, 0.53 \) and \( 0.58 \) fm). For each set we calculated the best \( V_0 \) and \( W_0 \)-values minimizing the \( \chi^2 \). Fig. 5.1 shows the agreement between the experimental data and their optical model best-fit obtained for \( r_0 = 1.24 \) fm and \( a_0 = 0.53 \) fm. All the results of this analysis are summarized in Table 5.1. As we can see all the sets provide a very good fit of the experimental data, as indicated by the similar \( \chi^2 \)-values listed in last column of Table 5.1.

<table>
<thead>
<tr>
<th>( r_0 = 1.20 ) fm</th>
<th>( a_0 ) (fm)</th>
<th>( V_0 ) (MeV)</th>
<th>( W_0 ) (MeV)</th>
<th>( \chi^2/pt )</th>
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</thead>
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<tr>
<td>0.43</td>
<td>242.3 ± 41.6</td>
<td>43.6 ± 14.8</td>
<td>0.509</td>
<td></td>
</tr>
<tr>
<td>0.48</td>
<td>156.1 ± 22.4</td>
<td>20.6 ± 6.8</td>
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<tr>
<td>0.53</td>
<td>111.6 ± 13.1</td>
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<tr>
<td>0.58</td>
<td>86.0 ± 8.4</td>
<td>5.9 ± 1.9</td>
<td>0.500</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( r_0 = 1.24 ) fm</th>
<th>( a_0 ) (fm)</th>
<th>( V_0 ) (MeV)</th>
<th>( W_0 ) (MeV)</th>
<th>( \chi^2/pt )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43</td>
<td>111.1 ± 18.8</td>
<td>20.1 ± 6.7</td>
<td>0.508</td>
<td></td>
</tr>
<tr>
<td>0.48</td>
<td>78.4 ± 10.6</td>
<td>10.3 ± 3.2</td>
<td>0.505</td>
<td></td>
</tr>
<tr>
<td>0.53</td>
<td>60.5 ± 6.2</td>
<td>5.7 ± 1.8</td>
<td>0.501</td>
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<tr>
<td>0.58</td>
<td>49.3 ± 2.5</td>
<td>3.4 ± 0.8</td>
<td>0.496</td>
<td></td>
</tr>
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Table 5.1: Upper (lower) panel: optical model parameters obtained from the best-fits of the \( ^{17}F \) scattering data at 90.4-MeV beam energy, fixing \( r_0 = 1.20 \) (1.24) \( \) fm and varying the diffuseness.

The fits were performed assuming that the collected data originated only from a pure elastic scattering process. In fact, as mentioned in Sec. 3.4, our energy resolution (3-4 \%, considering a limited number of strips) did not allow to solve possible contributions arising from inelastic excitations to the only \( ^{17}F \) excited state below the breakup threshold \( (E_x = 0.4593 \) MeV) and to the first \( ^{208}Pb \) excited level \( (E_x = 2.614 \) MeV).

This assumption was verified \textit{a posteriori} by running \textsc{Fresco} within the DWBA approach. For this purpose we used the potential parameters obtained from the previous best-fits, including also the possibility to excite to the first level both the projectile and the target. These excitations were
inserted into the input card of the code with their experimental $B(E\lambda)$ values, with $\lambda$ multipolarity of the transition. We used $B(E2) \uparrow = 21.64 \, \text{e}^2\text{fm}^4$ [Til93] for the $^{17}\text{F}$ excited state ($J^\pi = 1/2^+$) at $E_x = 0.4593 \, \text{MeV}$ and $B(E3) \uparrow = 6.1 \cdot 10^5 \, \text{e}^2\text{fm}^6$ [Mar86] for the $^{208}\text{Pb}$ first excited level ($J^\pi = 3^-$) at $E_x = 2.614 \, \text{MeV}$. Appendix A will briefly summarize the DWBA formalism (App. A.1) and will show a FRESCO input card used to estimate these inelastic transitions (App. A.2) and an input card used to minimize the $\chi^2$ between experimental data and theoretical predictions (App. A.3).

The results of the calculations are shown in Fig. 5.1. One can clearly see that, in this energy range, the contribution of the considered projectile excitation to the quasi-elastic cross section is at maximum $\sim 2\%$ and for this reason it could be neglected in first approximation. Also the contribution arising from the target excitation was found to be negligible, since in this angular range it is always lower than $1\%$ with respect to the quasi-elastic scattering cross section.

### 5.1.2 $^{17}\text{F} + ^{208}\text{Pb}$ system at 98-MeV beam energy

![Angular distribution graph](image)

**Figure 5.2:** *Experimental angular distribution for the scattering process in the $^{17}\text{F} + ^{208}\text{Pb}$ system at 98-MeV beam energy.* The continuous dark line represents the optical model best-fit of the experimental data with $V_0 = 60.5 \, \text{MeV}$, $r_0 = 1.24 \, \text{fm}$ and $a_0 = 0.53 \, \text{fm}$. The red curve considers also the possibility to excite the first $^{17}\text{F}$ excited level at 0.4953 MeV and the blue one includes also the excitation to the first $^{208}\text{Pb}$ state.
The influence of the $^{17}$F beam energy on the potential parameters has been investigated analyzing, within the same formalism, the scattering data for the $^{17}$F + $^{208}$Pb system also at 98-MeV beam energy.

This experiment was carried out at the Holyfield Radioactive Ion Beam Facility (HRIBF) of the Oak Ridge National Laboratory (ORNL, Tennessee, USA) by part of our collaboration [Lia03]. In this case the short-lived $^{17}$F nuclei were produced through the ISOL method via the $^{16}$O($d,n)^{17}$F reaction [Wel99]. Fig. 5.2 shows the $^{17}$F + $^{208}$Pb scattering angular distribution at 98-MeV beam energy. In spite of some oscillations, the trend of the experimental data is rather flat up to $\theta_{cm} \sim 100^\circ$ and then it slowly decreases. The errors plotted in the figure include both statistical and systematical errors.

For this analysis we adopted two fitting procedures, with the same $r_0$ and $a_0$ grids previously used:

1. we fixed the real part of the potential at the value obtained from our previous analysis at 90.4-MeV beam energy and varied only $W_0$ in order to minimize the $\chi^2$ (Procedure I);

2. we varied both $V_0$ and $W_0$, using the previously calculated values as starting points (Procedure II).

The results of the two procedures are listed in Tables 5.2 and 5.3. Also in this case the experimental data were analyzed assuming that they originate from a “pure” elastic scattering process. The possibility to excite the first excited state both in $^{17}$F and in $^{208}$Pb was investigated using the DWBA approach. It was seen that in the angular range of the experiment ($60^\circ \leq \theta_{cm} \leq 120^\circ$) the contributions of these inelastic excitations to the quasi-elastic cross section were at maximum $\sim 5\%$ (see Fig. 5.2) and they were therefore neglected.

Comparison between Tables 5.2 and 5.3 shows that the two procedures gave slightly different results. For each fit performed with the second approach, the depth of the real part of the potential $V_0$ resulted to be smaller than that in Procedure I, while $W_0$ shows the opposite trend. The maximum variations between the two evaluations are $\sim 15\%$ for $V_0$ and $\sim 25\%$ for $W_0$. These deviations given, it is very interesting to compare the best-fit parameters obtained at 90.4 MeV and at 98 MeV. We clearly see from Tables 5.1 and 5.3 that, within the small differences already pointed out, for each fit the depth of the real Woods-Saxon well is practically the same below and above the Coulomb barrier. On the other side, $W_0$ gets around two/three times larger going from 90.4- to 98-MeV beam energy. This is quite reasonable since the effects of non-elastic channels, dominating the imaginary part of the nuclear potential, are much larger at energies above the Coulomb barrier.
<table>
<thead>
<tr>
<th>$a_0$ (fm)</th>
<th>$V_0$ (MeV) (fixed)</th>
<th>$W_0$ (MeV)</th>
<th>$\chi^2/\nu$</th>
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</thead>
<tbody>
<tr>
<td>0.43</td>
<td>242.3</td>
<td>83.25 ± 0.01</td>
<td>0.843</td>
</tr>
<tr>
<td>0.48</td>
<td>156.1</td>
<td>42.25 ± 0.01</td>
<td>0.882</td>
</tr>
<tr>
<td>0.53</td>
<td>111.6</td>
<td>23.81 ± 0.01</td>
<td>0.960</td>
</tr>
<tr>
<td>0.58</td>
<td>86.0</td>
<td>14.52 ± 0.01</td>
<td>1.099</td>
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</table>

Table 5.2: Upper (lower) panel: optical model parameters obtained from the best-fits of the scattering data for the system $^{17}F + ^{208}Pb$ at 98-MeV beam energy as a function of the diffuseness $a_0$. For each fit $V_0$ was fixed at the value obtained from the analysis at 90.4-MeV beam energy and $r_0$ at 1.20 (1.24) fm (Procedure I).

<table>
<thead>
<tr>
<th>$a_0$ (fm)</th>
<th>$V_0$ (MeV)</th>
<th>$W_0$ (MeV)</th>
<th>$\chi^2/\nu$</th>
</tr>
</thead>
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<td>240.4 ± 33.1</td>
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</tr>
<tr>
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<td>29.1 ± 5.8</td>
<td>0.891</td>
</tr>
<tr>
<td>0.58</td>
<td>73.2 ± 6.6</td>
<td>19.1 ± 3.3</td>
<td>0.981</td>
</tr>
</tbody>
</table>

Table 5.3: Upper (lower) panel: optical model parameters obtained from the best-fits of the $^{17}F$ scattering data at 98-MeV beam energy, fixing $r_0 = 1.20$ (1.24) fm and varying the diffuseness. For each fit the values obtained for the analysis at 90.4 MeV and listed in Table 5.1 were used as starting points for $V_0$ and $W_0$ (Procedure II).
5.1.3 \( ^{11}\text{Be} + ^{209}\text{Bi} \) system

A coherent optical model analysis has been carried out also for the \( ^{11}\text{Be} \) ions scattered from a \( ^{209}\text{Bi} \) target. As discussed in details in Ch. 4, the \( ^{11}\text{Be} \) secondary beam produced by the RIPS facility at RIKEN had a continuous energy spectrum with a total spread of \( \sim 15 \text{ MeV} \) (see Fig. 4.9). In order to get enough statistics the beam energy range was divided into 2.0-MeV energy bins, with average values ranging from 40 to 48 MeV.

Also for this system the nuclear interaction between the target and the projectile has been described through Woods-Saxon wells for both real and imaginary part of the potential. We used the code FRESCO to provide a best-fit analysis of the experimental data and to get the six parameters needed to define the depth, the radius and the diffuseness of each potential well. Since the data accuracy was not as good as for the system \( ^{17}\text{F} + ^{208}\text{Pb} \), two procedures have been followed to calculate the starting point or, in case, the fixed value of each parameter. In the first approach we used the parameters obtained from the Akyüz-Winther potential, whereas in the second one we exploited the similarities between the elastic scattering angular distributions for the systems \( ^{9,11}\text{Be} + ^{209}\text{Bi} \) in this energy range.

First method: Akyüz-Winther potential

In this procedure all the parameters were calculated from the Akyüz-Winther potential [Bro91]. This approach was undertaken since it has an excellent agreement with a large amount of experimental data over a wide range of masses and energies, at least for nuclei close to valley of stability. In this framework the depth of the real Woods-Saxon well is determined with the following equation:

\[
V_0 = 16\pi \gamma \tilde{R} a_0 \tag{5.2}
\]

where the parameters \( \gamma \), \( \tilde{R} \) and the diffuseness \( a_0 \) are given by:

\[
\tilde{R} = \frac{R_P R_T}{R_P + R_T} \tag{5.3}
\]

\[
\gamma = \gamma_0 \left[ 1 - k \left( \frac{N_P - Z_P}{A_P} \right) \left( \frac{N_T - Z_T}{A_T} \right) \right] \tag{5.4}
\]

\[
a_0 = 0.63 \text{ fm} \tag{5.5}
\]

with \( R_{P(T)}, N_{P(T)}, Z_{P(T)} \) and \( A_{P(T)} \) being the radius, the neutron, the atomic and the mass number of the projectile (target), respectively, \( \gamma_0 = 0.95 \text{ MeV} \).
fm$^{-2}$ and $k = 1.8$. Within this formalism, the nuclear radius is given by:

$$R = 1.233 A^{1/3} - 0.98 A^{-1/3},$$

(5.6)

Last formula was adopted only for the $^{209}$Bi target, while in order to take into account the halo structure of the $^{11}$Be nucleus we used the usual formula $R_P = r_0 A^{-1/3}$ with $r_0 = 1.4$ fm [Tan85] instead of the well-known $r_0 = 1.2$ fm. Once we determined the nuclear radii, we can easily get the radius parameter to be inserted in the code Fresco to describe the width of the Woods-Saxon wells, $r_0$:

$$r_0 = \frac{R_0}{(A_P^{1/3} + A_T^{1/3})} = \frac{R_P + R_T + 0.29}{(A_P^{1/3} + A_T^{1/3})}$$

(5.7)

The resulting starting points for the potential parameters used in the optical model best-fits of the experimental data were: $V_0 = W_0 = 58.658$ MeV for the depths, $r_{0w} = r_{0ve} = r_0 = 1.294$ fm for the radii and $a_v = a_w = a_0 = 0.63$ fm for the diffusenesses.

<table>
<thead>
<tr>
<th>E (MeV)</th>
<th>$V_0$ (MeV) (fixed)</th>
<th>$W_0$ (MeV)</th>
<th>$\chi^2$/pt</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>58.658</td>
<td>13.43 ± 0.01</td>
<td>3.609</td>
</tr>
<tr>
<td>42</td>
<td>58.658</td>
<td>20.48 ± 0.01</td>
<td>10.652</td>
</tr>
<tr>
<td>44</td>
<td>58.658</td>
<td>40.78 ± 0.01</td>
<td>14.023</td>
</tr>
<tr>
<td>46</td>
<td>58.658</td>
<td>84.76 ± 0.01</td>
<td>12.670</td>
</tr>
<tr>
<td>48</td>
<td>58.658</td>
<td>144.79 ± 0.01</td>
<td>10.443</td>
</tr>
</tbody>
</table>

Table 5.4: Optical model parameters obtained from the best-fits of the $^{11}$Be elastic scattering angular distribution from 40- to 48-MeV beam energy. The values have been calculated fixing $V_0 = 58.658$, $r_0 = 1.294$ fm, $a_0 = 0.63$ fm and varying the imaginary depth $W_0$.

At each energy we fixed all the parameters to the calculated values and only $W_0$ could vary in order to minimize the $\chi^2$. Our experimental energy resolution did not allow to distinguish between “pure” elastic scattering events and quasi-elastic events with the only $^{11}$Be excited state below the breakup threshold ($J^\pi = 1/2^-$, $E_x = 0.320$ MeV) populated in the interaction. This process has been studied within both the DWBA approach and the Coupled-Channel formalism, including in the code Fresco the possibility to excite the first $^{11}$Be excited state from the ground state with $B(E1) \uparrow = 0.115$ e$^2$fm$^2$ [Ajz90]. As a function of the incident energy, its contribution to the quasi-elastic cross section was found to range at maximum from 21% (at 40 MeV) to 32% (at 46 MeV). Therefore this excitation was properly taken into
Figure 5.3: Experimental angular distributions and optical model best-fits for the $^{11}\text{Be}$ scattering process from a $^{209}\text{Bi}$ target in the energy range from 40 to 48 MeV. The continuous dark lines are the best-fits of the experimental data performed assuming a “pure” elastic scattering process, while red curves take also into account the possibility to excite the unsolved first $^{11}\text{Be}$ excited level at 0.320 MeV.

account in the theoretical calculations. Fig. 5.3 shows the results of the optical model best-fits: dark lines consider only the elastic scattering process, while red ones consider also the excitation to the first $^{11}\text{Be}$ level. Table 5.4 lists the best-fit parameters at all the energies.

We also analyzed the possibility to excite during a scattering process the following “unsolved” $^{209}\text{Bi}$ excited states: the first level $J^\pi = 7/2^-$ at $E_x = 0.896$ MeV, the second one $J^\pi = 13/2^-$ at $E_x = 1.609$ MeV and the multiplet [3$^-$ $\otimes h^{7/2}$] at $E_x \sim 2.6$ MeV [Mar91]. These contributions were found to be very small ($\ll 1.0\%$) in comparison with the quasi-elastic cross section and were not included in the calculations.
As we can see from Fig. 5.3 a remarkable agreement between experimental data and the best-fit analysis was reached only at 40-MeV beam energy, while at higher energies the theoretical curves systematically underestimate the experimental data. We investigated the origin of such a difference performing some test runs to check the influence of each potential parameter. We saw that the main deviation arises from the nuclear radius parameters used to reproduce the $^{11}$Be halo structure. This means that a larger flux into non-elastic channels is expected from a theoretical point-of-view, due to the projectile halo structure. This reduction has not been experimentally observed since the angular distributions for the $^{11}$Be scattering process from a $^{209}$Bi target resulted to be very similar to those measured for the system $^9$Be + $^{209}$Bi in the same energy range [Sig00].

From this analysis, we can conclude that the effects related to low binding energy of both the Beryllium isotopes (i.e. mainly the possibility that a breakup process could occur) are larger than those arising from the $^{11}$Be halo structure (namely a reduced Coulomb barrier and consequently a higher probability for this nucleus to fuse with target with respect to the $^9$Be ion).

**Second method: $^9$Be approach**

In the second approach our starting point was the similarities between the scattering differential cross sections for the systems $^{11}$Be + $^{209}$Bi and $^9$Be + $^{209}$Bi [Sig00] in the energy range around the Coulomb barrier. As shown in Fig. 5.4, up to 44-MeV beam energy we can see that the scattering angular distributions for the two systems are rather similar at all measured angles, within the experimental errors. This analogy persists also at higher energies but only for scattering angles smaller than $\sim 110^\circ$. An analogous behavior of these two systems has already been observed in this energy range also for the fusion cross sections [Sig04].

For this reason, we used the potential parameters obtained for the $^9$Be scattering process from a $^{209}$Bi target [Sig00] as starting points for the new analysis. In this way the $^{11}$Be ion has been described in our code by the same standard as $^9$Be, completely forgetting its halo structure. The fits of the experimental data were performed fixing all the parameters, except the depth of the imaginary part of the potential $W_0$. Also in this case we did a double fit of the angular distributions in order to estimate the influence of the unsolved excitation to the $^{11}$Be first level. Its contribution to the quasi-elastic cross section was found to be as large as in the previous approach: from 20% at 40 MeV to 32% at 48 MeV and therefore this inelastic process was properly considered in the calculations. The results of all the best-fit analyses are shown in Fig. 5.4 and the resulting potential parameters are
5.1. Optical model analysis

Figure 5.4: Experimental angular distributions for the scattering process in the systems $^9$Be + $^{209}$Bi from 40- to 48-MeV beam energy. The continuous dark lines are the best-fits of the $^{11}$Be experimental data performed for a “pure” elastic scattering process, while red curves consider also the excitation to the first $^{11}$Be excited level at 0.320 MeV. For each fit we used the potential parameters obtained for the system $^9$Be + $^{209}$Bi as starting points for those of the $^{11}$Be system.

listed in Table 5.5.

Comparing the results obtained with the two methods it is evident that a better agreement has been achieved in the second case, as also shown by $\chi^2$ test (last column of Tables 5.4 and 5.5). In particular, up to 44-MeV beam energy and for scattering angles larger than 110°, the cross sections are much better reproduced by the second approach. At higher energy the experimental points collected at backward angles are still above the fitting curves. These deviations could arise from a systematic error in the evaluation of these cross sections. In fact, the assumption done for normalization
Table 5.5: Optical model parameters obtained from the best-fits of the $^{11}\text{Be}$ elastic scattering angular distribution from 40- to 48-MeV beam energy. The values have been calculated fixing all the parameters to those obtained for the system $^9\text{Be} + ^{209}\text{Bi}$ and varying only the imaginary depth $W_0$ with starting points listed in the column labeled "$W_0$ ($^9\text{Be}$)". $r_{0\omega} = 1.178$ fm and $a_\omega = 0.63$ fm for all the energies.

purposes (see Sec. 4.6.2) that at 38 MeV the measured cross section was purely Rutherford could generate a systematic overestimation of the cross sections, in particular for large $\theta$.

However, we have to point out that the uncertainties due to the reconstruction and the binning of the incident energy could largely affect the quality of the resulting scattering angular distributions. Therefore, a better energy resolution of the secondary beam is needed to provide a more accurate measurement of the differential cross sections for this process.

5.2 Strong absorption radii

The optical model best-fit analysis described in the previous Section showed that in the energy range around the Coulomb barrier, the fitting procedure strongly depends on the starting points used for the potential parameters and on the choice between free and fixed parameters. Therefore such an analysis does not usually allow to determine a unique set of parameters for the nuclear potential between the interacting nuclei. However both the real and the imaginary part of the potential evaluated at the strong absorption radius (the distance roughly corresponding to the classical touching point of the colliding nuclei) are well defined and do not depend on the fitting ambiguities.

The collected data allows to determine the strong absorption radii, to evaluate the nuclear potential at those radial points for the system $^{17}\text{F} + ^{208}\text{Pb}$ at 90.4- and 98-MeV beam energies and to compare these values with
those obtained for the reaction involving the stable Fluorine isotope $^{19}$F + $^{208}$Pb at the same colliding energies [Lin01].

### 5.2.1 $^{17}$F + $^{208}$Pb system at 90.4-MeV beam energy

![Graph](image)

Figure 5.5: Behavior of real and imaginary potential fitting the $^{17}$F elastic scattering data at 90.4 MeV for the different values of diffuseness and radius listed in Table 5.1.

From the optical model analysis performed in Sec. 5.1.1 we deduced both the real and the imaginary strong absorption radii for the system $^{17}$F + $^{208}$Pb at 90.4-MeV beam energy: $R_{SV} = 11.97 \pm 0.14$ fm and $R_{SW} = 13.53 \pm 0.19$ fm. The potential parameter values used for these evaluation are listed in Table 5.1. The potentials at these radial points resulted to be $V_{sa} = 3.84 \pm 0.51$ MeV and $W_{sa} = 0.020 \pm 0.002$ MeV, respectively. Fig. 5.5 shows the dependence of $V(r)$ and $W(r)$ from the radius $r_0$ and the diffuseness $a_0$ parameters, in the ranges around $R_{SV}$ and $R_{SW}$.

$R_{SV}$ and $R_{SW}$ are quite different from each other. In particular the imaginary part of the nuclear potential is extended well outside the nucleus core. This scenario is completely different for the stable isotope $^{19}$F. In fact for the system $^{19}$F + $^{208}$Pb at 91-MeV beam energy, which has the same $E_{cm}/V_C$ ratio as the system $^{17}$F + $^{208}$Pb at 90.4 MeV, the real and imaginary
strong absorption radii, reported in [Lin01] are very similar \( R_{SV} = 12.32 \text{ fm} \) and \( R_{SW} = 12.12 \text{ fm} \).

### 5.2.2 \(^{17}F + ^{208}\text{Pb}\) system at 98-MeV beam energy

![Figure 5.6: Real and imaginary potential dependence from the diffuseness and radius parameters used to fit the \(^{17}F\) elastic scattering data at 98-MeV beam energy. The best-fit values are listed in Table 5.3.](image)

The procedures followed in Sec. 5.1.2 to fit the scattering data for the \(^{17}F + ^{208}\text{Pb}\) reaction at 98-MeV beam energy allowed to determine also in this case the real and the imaginary strong absorption radii. We remind that two methods have been adopted for this analysis. In the first one (Procedure I) the depth of the real part of the potential was fixed at the value obtained from the best-fit analysis at 90.4-MeV beam energy and only \( W_0 \) was free to vary in order to minimize the \( \chi^2 \). In the second procedure (Procedure II) both \( V_0 \) and \( W_0 \) could vary and the calculated values at lower energy were used as starting points. Fig. 5.6 shows the results obtained with the second approach.

The two procedures gave slightly different results, though leading to the same conclusions. From the first approach, \( R_{SV} \) and \( V_{sa} \) were the same as at lower energies, since all the used real potential parameters were those...
obtained at 90.4 MeV. For the other component of the nuclear interaction, an imaginary strong absorption radius \( R_{SW} = 13.08 \pm 0.03 \text{ fm} \) has been estimated and the corresponding potential resulted to be \( W_{sa} = 0.102 \pm 0.007 \text{ MeV} \).

From the second analysis, we obtained \( R_{SV} = 12.20 \pm 0.08 \text{ fm} \) and \( R_{SW} = 12.67 \pm 0.18 \text{ fm} \). The values of the two potentials at these points turned out to be \( V_{sa} = 2.23 \pm 0.09 \text{ MeV} \) and \( W_{sa} = 0.27 \pm 0.03 \text{ MeV} \). Table 5.6 summarizes all the strong absorption radii evaluated and the corresponding potentials as well as those obtained for the system \(^{19}\text{F} + ^{208}\text{Pb}\) [Lin01].

<table>
<thead>
<tr>
<th>( R_{SV} ) (fm)</th>
<th>( V_{sa} ) (MeV)</th>
<th>( R_{SW} ) (fm)</th>
<th>( W_{sa} ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{17}\text{F} + ^{208}\text{Pb} )</td>
<td>( ^{19}\text{F} + ^{208}\text{Pb} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90.4</td>
<td>11.97 ± 0.14</td>
<td>3.84 ± 0.51</td>
<td>13.53 ± 0.19</td>
</tr>
<tr>
<td>98 (I)</td>
<td>11.97 ± 0.14</td>
<td>3.84 ± 0.51</td>
<td>13.08 ± 0.03</td>
</tr>
<tr>
<td>98 (II)</td>
<td>12.20 ± 0.03</td>
<td>2.23 ± 0.09</td>
<td>12.67 ± 0.18</td>
</tr>
</tbody>
</table>

Table 5.6: Upper (lower) panel: real and imaginary strong absorption radii and corresponding nuclear potentials for the system \(^{17}\text{F}^{19}\text{F}) + ^{208}\text{Pb}\) at 90.4- (91- ) and 98- MeV beam energy. The line labeled “98 (I)” and “98 (II)” refers to values obtained at 98 MeV using the Procedure I and Procedure II, respectively, see text for additional details.

From the comparison we can see a trend of \( R_{SW} \): the \(^{17}\text{F}\) imaginary potential results to extend well outside the core, which is not the case for \(^{19}\text{F}\) (\( R_{SV} \approx R_{SW} \)). This effect might be related to the very low binding energy of the last proton in \(^{17}\text{F}\).

### 5.3 Reaction cross sections

Last information extracted from the optical model analysis was the reaction cross section. This quantity, as the strong absorption radii, is rather independent from the potential parameters used to fit the elastic scattering angular distributions once that a sufficient agreement between experimental data and theoretical predictions has been achieved.
In this Section we will present the reaction cross sections evaluated for the systems $^{17}\text{F} + ^{208}\text{Pb}$ and $^{11}\text{Be} + ^{209}\text{Bi}$ and those obtained in other experiments for similar mass systems ($^{19}\text{F}, ^{16,17}\text{O} + ^{208}\text{Pb}$ and $^9\text{Be} + ^{209}\text{Bi}$) in the energy range around the Coulomb barrier.

### 5.3.1 $^{17,19}\text{F}, ^{16,17}\text{O} + ^{208}\text{Pb}$ reaction cross sections

The reaction cross sections have been deduced from the scattering differential cross sections (see Fig. 5.7) of four similar mass systems:

1. $^{16}\text{O} + ^{208}\text{Pb}$ ($S_n = 7.162$ MeV and $S_p = 12.127$ MeV) at 78-MeV beam energy [Tho89];

2. $^{17}\text{O} + ^{208}\text{Pb}$ ($S_n = 4.143$ MeV) at 78 MeV [Lil87];

3. $^{17}\text{F} + ^{208}\text{Pb}$ ($S_p = 0.601$ MeV) at 90.4 MeV [Rom04];

4. $^{19}\text{F} + ^{208}\text{Pb}$ ($S_n = 4.014$ MeV and $S_p = 7.994$ MeV) at 91-MeV beam energy [Lin01].

At these energies, the ratios $E_{cm}/V_C$ are almost the same for all the systems, as reported in Table 5.7. This Table summarizes the parameters used to fit the scattering angular distributions for the four systems and its last column lists the values of the deduced reaction cross sections. The data for the reaction $^{19}\text{F} + ^{208}\text{Pb}$ have been reanalyzed using the code Fresco following the same procedure as for $^{17}\text{F}$ and the results are slightly different with respect to the published data [Lin01]. Table 5.7 reports the values obtained from our analysis.

<table>
<thead>
<tr>
<th></th>
<th>$E_{lab}$ (MeV)</th>
<th>$E_{cm}/V_C$</th>
<th>$V_0$ (MeV)</th>
<th>$r_{0v}$ (fm)</th>
<th>$a_v$ (fm)</th>
<th>$W_0$ (MeV)</th>
<th>$r_{0w}$ (fm)</th>
<th>$a_w$ (fm)</th>
<th>$\sigma_R$ (mb)</th>
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</thead>
<tbody>
<tr>
<td>$^{17}\text{F}$</td>
<td>90.4</td>
<td>0.96</td>
<td>60.5</td>
<td>1.24</td>
<td>0.53</td>
<td>5.7</td>
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<td>0.96</td>
<td>107.6</td>
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<td>0.53</td>
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<td>1.24</td>
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<tr>
<td>$^{16}\text{O}$</td>
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<td>0.93</td>
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<tr>
<td>$^{17}\text{O}$</td>
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<td>82.8</td>
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<td>9.9</td>
<td>1.23</td>
<td>0.60</td>
<td>91</td>
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</table>

Table 5.7: Optical model potential parameters obtained from the best-fits of four different reactions. The data for Fluorine isotopes arise from our analysis, while the data for Oxygen isotopes are taken from [Tho89] and [Lil87], respectively. The target is $^{208}\text{Pb}$ for all the systems.
Figure 5.7: Comparison of the optical model analysis of the experimental elastic scattering angular distributions for four similar mass systems: $^{16,17}O + ^{208}Pb$ at 78 MeV, $^{17}F + ^{208}Pb$ at 90.4 MeV and $^{19}F + ^{208}Pb$ at 91-MeV beam energy. The potential parameters used are indicated in Table 5.7. For the reaction involving $^{19}F$, a standard Woods-Saxon potential was used, adding the excitation to the second excited state at $E_x = 0.197$ MeV from DWBA and Coupled-Channel calculations (the first level at $E_x = 0.110$ MeV was not included, since its coupling to the ground state is very weak). The curve labeled “elastic + 2nd state inelastic scattering best-fit” is the result of this calculation. From this analysis, it was also possible to extract the pure elastic data (the curve labeled “optical model elastic scattering analysis”). Experimental data for the system $^{17}F + ^{208}Pb$ are shown for completeness.

Fig. 5.7 shows that the $^{17}F$ behavior is much more similar to that of the Oxygen isotopes rather than to $^{19}F$ one. This fact is quite surprising, since the radioactive and very weakly bound $^{17}F$ was expected to behave differently from the well-bound $^{16}O$ and $^{17}O$.

The deduced reaction cross sections for the $^{16,17}O$ and $^{17}F$ projectiles are about a factor 3 smaller than the $^{19}F$ case. This large difference could originate from the collective structure of $^{19}F$, since its ground state ($J^\pi = \frac{1}{2}^+$) is very strongly coupled to the excited state at $E_x = 0.197$ MeV, ($J^\pi = \frac{5}{2}^+$), belonging to the same rotational band [Til95]. This is in agreement with the theoretical descriptions. We performed both Coupled-Channel and DWBA
calculations for the excitations to this (collective) state in $^{19}$F as well as to the $^{17}$F first (single-particle) excited level. For $^{19}$F ($^{17}$F), the contribution of the calculated inelastic scattering cross section to the elastic scattering angular distribution is fairly large (small) [$\leq 40\%$ (2\%)]. Therefore, we can conclude that at energies below the Coulomb barrier, it seems to be easier to excite collective structures (even in a well-bound nucleus like $^{19}$F) than to breakup a very loosely bound nucleus, as $^{17}$F. We have to bear in mind that in the $^{19}$F case the low excitation energy as well as the strong transition probability B(E2) enhances the process.

![Graph](image)

**Figure 5.8:** Reaction cross section for the systems $^{17,19}$F + $^{208}$Pb in the energy range between 90 and 120 MeV. For a better comparison between the two systems data have been divided by $R^2$, with $R$ sum of projectile and target radius, and plotted versus $E_{cm}/V_c$, with $V_c$ Coulomb barrier. We used $V_c = 86.86$ MeV (85.88 MeV) for the system $^{17}$F ($^{19}$F) + $^{208}$Pb. The values at 91 and 98 MeV ($E_{cm}/V_c \sim 0.96$ and 1.04, respectively) for both reactions and at 120 MeV ($E_{cm}/V_c \sim 1.28$) for the reaction involving the $^{17}$F ion have been deduced from optical model best-fit analysis. The other values are theoretical predictions, see text for additional details.

The reaction cross sections have also been deduced from the experimental scattering angular distributions for the systems $^{17,19}$F + $^{208}$Pb at energies above the Coulomb barrier (see Fig. 5.8). In particular, we evaluated the reaction cross sections at 98 MeV for both systems [Lia03, Lin01] and at 120-MeV beam energy only for the lighter isotope [Lia03]. Some calculations have also been performed at the intermediate energy of 110 MeV for both
systems and at 120 MeV for the stable Fluorine isotope, where the scattering cross sections have not been measured yet.

For the system \(^{17}\text{F} + ^{208}\text{Pb}\) at 110-MeV beam energy we used the values obtained from previous best-fit analysis for all the potential parameters except for the depth of the imaginary potential, where we assumed \(W_0 = 45.0\) MeV, average value of the \(W_0\)-values obtained at 98 and 120 MeV. On the other side, for the reaction \(^{19}\text{F} + ^{208}\text{Pb}\) at 110 MeV and 120 MeV we also fixed all the parameters at the values previously obtained, while for \(W_0\) we proceeded as for \(^{17}\text{F}\). In fact, \(W_0\) increases of about 10 MeV moving from \(\sim 91\)- to 98-MeV beam energy for both reactions. For this reason we fixed \(W_0 = 62.2\) MeV (87.2 MeV) at 110-MeV (120-MeV) beam energy, 30 MeV (55 MeV) larger than the \(W_0\)-value at 98 MeV, as for the reaction involving the \(^{17}\text{F}\) ion. Table 5.8 summarizes all the potential parameters used to evaluate the reaction cross sections for both systems (theoretical points have been marked with an asterisk (*)�).

<table>
<thead>
<tr>
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</tr>
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<th>(19\text{F} + ^{208}\text{Pb})</th>
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<tr>
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<td>110</td>
<td>107.6</td>
</tr>
<tr>
<td>120</td>
<td>107.6</td>
</tr>
</tbody>
</table>

Table 5.8: Upper (lower) panel: real and imaginary strong absorption radii and corresponding nuclear potentials for the system \(^{17}\text{F}(^{19}\text{F}) + ^{208}\text{Pb}\) for beam energies in the range between 90-120 MeV. The lines labeled “98 (I) (II)” refers to values obtained at 98 MeV using Procedure I and II, respectively, (see Sec. 5.1.2). See text for additional details on the \(W_0\)-values marked with (*)�).
Fig. 5.8 shows the reaction cross section for the systems $^{17,19}\text{F} + ^{208}\text{Pb}$ in the energy range 90-120 MeV. For a better comparison between the two systems, the reaction cross sections have been divided by the $R^2$, with $R$ sum of projectile and target radius, and plotted versus $E_{cm}/V_C$, with $V_C$ Coulomb barrier. As we can see the reaction cross section results to be larger for the $^{19}\text{F}$ ions than for $^{17}\text{F}$ nuclei especially at low energies. This suggests that even at energies above the Coulomb barrier to excite $^{19}\text{F}$ collective structure seems to be easier than $^{17}\text{F}$ single-particle states with consequently break of the scattered ion. Of course this finding needs to be confirmed by experiments at 110 and 120 MeV.

### 5.3.2 $^{9,11}\text{Be} + ^{209}\text{Bi}$ Reaction Cross Section

Evaluation of the reaction cross section for the system $^{11}\text{Be} + ^{209}\text{Bi}$ at the Coulomb barrier is based on the optical model best-fit analysis described in Sec. 5.1.3. We remind that two different approaches were undertaken to reproduce the experimental data: the first using the Akyüz-Winther potential and inserting a priori the $^{11}\text{Be}$ halo structure, the second exploiting the similarities between the scattering differential cross sections for the systems $^{9,11}\text{Be} + ^{209}\text{Bi}$. The average difference between the two methods resulted to be about 20-25%. Fig. 5.9 and Table 5.9 show the reaction cross sections obtained with both methods and those deduced for the system $^{9}\text{Be} + ^{209}\text{Bi}$ [Sig00]. For a better comparison the reaction cross sections have been plotted in the Figure as $\sigma_R/R^2$, with $R$ sum of $^{11}\text{Be}$ and $^{209}\text{Bi}$ nuclear radii, versus the ratio $E_{cm}/V_C$. Since a better agreement with the experimental data was achieved using Procedure II, in the following we will refer only to this set of the data.

Comparison between the two Beryllium isotopes in interaction with a $^{209}\text{Bi}$ target shows a rather similar trend of the reaction cross sections at Coulomb barrier energies. Even if the scattering angular distributions for the two systems are quite similar (at least up to 44-MeV beam energy), the reaction cross sections for $^{11}\text{Be}$ results to be $\sim 50\%$ larger than for $^9\text{Be}$. This discrepancy should originate from the fitting procedure adopted in the optical model analysis, since the experimental values at large scattering angles in the $^{11}\text{Be}$ case were generally underestimated. However, the comparison between the scattering angular distributions suggests that in this energy range the reaction cross sections for the system $^{11}\text{Be} + ^{209}\text{Bi}$ should be at least as large as for the the system $^9\text{Be} + ^{209}\text{Bi}$. Therefore we can conclude that at Coulomb barrier energies the effects of the $^{11}\text{Be}$ lower binding energy and halo structure on the elastic scattering process seem to be larger, within the experimental accuracy, than the probability to excite the $^9\text{Be}$ rotational band.
Figure 5.9: Reaction cross sections for the systems $^{9,11}\text{Be} + ^{209}\text{Bi}$ at Coulomb barrier energies. For an optimal comparison between the two systems, data have been divided by $R^2$, with $R$ sum of the radii of the projectile and the target, and plotted versus $E_{cm}/V_C$, with $V_C$ Coulomb barrier. $V_C = 38.4$ MeV [Sig99] (37.7 MeV) was used for the system $^9\text{Be} + ^{11}\text{Be} + ^{209}\text{Bi}$. Two evaluations have been plotted for the system $^{11}\text{Be} + ^{209}\text{Bi}$, according to the two best-fit procedures adopted in the optical model analysis. See text for more details.

<table>
<thead>
<tr>
<th>$E$ (MeV)</th>
<th>$^{11}\text{Be} + ^{209}\text{Bi}$</th>
<th>$^9\text{Be} + ^{209}\text{Bi}$</th>
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<tbody>
<tr>
<td>$\sigma_R$ (I) (mb)</td>
<td>$\sigma_R$ (II) (mb)</td>
<td>$\sigma_R$ (mb)</td>
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<td>48</td>
<td>1791</td>
<td>1385</td>
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</tbody>
</table>

Table 5.9: Reaction cross sections for the systems $^{9,11}\text{Be} + ^{209}\text{Bi}$ at the Coulomb barrier. Column (I): $\sigma_R$ evaluated using the best-fit parameters listed in Table 5.4; column (II): reaction cross sections deduced from the potential parameter set of Table 5.5.

Based on a $K = 3/2^-$ g.s. [Sig01a].
Chapter 6

Discussion

This Chapter will present a brief overview of the present status-of-the-art for the two systems studied in this thesis, $^{17}\text{F} + ^{208}\text{Pb}$ and $^{11}\text{Be} + ^{209}\text{Bi}$, in the energy range around the Coulomb barrier. We will focus our attention on the interplay among all the open reaction channels and in particular the influences of the projectile low binding energy and halo structure on the fusion process. Although more accurate data would be necessary to get a deeper understanding of the whole interaction mechanism, present data are sufficient to outline a scenario of the main reaction mechanisms and constitute a first important insight toward a systematic analysis of the behavior of loosely bound radioactive beams.

6.1 The system $^{17}\text{F} + ^{208}\text{Pb}$

The available experimental data at Coulomb barrier energies for the system $^{17}\text{F} + ^{208}\text{Pb}$ include:

- **scattering** cross sections at 90.4 MeV [Rom04], at 98 MeV and 120 MeV [Lia03];

- **fusion** cross section data from 87 MeV to 99 MeV [Reh98];

- **exclusive breakup** $^{17}\text{F} \rightarrow ^{16}\text{O} + p$ data at 90.4 MeV [Rom04];

- **inclusive breakup** $^{17}\text{F} + ^{208}\text{Pb} \rightarrow ^{16}\text{O} + (p + ^{208}\text{Pb})$ measurements at 98 and 120 MeV [Lia03].

Fig. 6.1 shows the experimental cross sections for fusion and breakup processes and the reaction cross section deduced from elastic scattering measurements.
Figure 6.1: Experimental cross sections for breakup and fusion processes and reaction cross section for the system $^{17}$F + $^{208}$Pb at Coulomb barrier energies. A contribution from the incomplete fusion of $^{16}$O to the fusion cross section cannot be excluded a priori (see text for more details). Theoretical data come from [Lia03].

All the measured cross sections suffer of the typical uncertainties related to the experiments with first generation RIB’s. In particular, for the scattering experiments, both beam and detector energy resolutions were not enough to separate the quasi-elastic events from the pure elastic ones. However, theoretical evaluations of the inelastic scattering cross section to the first $^{17}$F excited state at $E_x = 0.4953$ MeV give a negligible contribution with respect to the pure elastic cross section. The exclusive breakup cross section was extrapolated from the differential cross section measured at only one backward angle ($\bar{\theta} = 125^\circ$) [Rom04]. Finally, fusion cross section was deduced from fission fragments yield [Reh98] and we cannot a priori exclude a contribution from incomplete fusion of $^{16}$O, following $^{17}$F breakup into $^{16}$O + p.

Fig. 6.1 reports also the theoretical calculations for the inclusive and exclusive breakup [Lia03]. These are the results of dynamical calculations, where the relative motion of the two fragments is treated quantum mechanically by solving the time-dependent Schrödinger equation for the two-body breakup process in the Coulomb and nuclear field of the target [Esb95]. The curve relative to exclusive breakup is drawn also to suggest a possible trend for the cross section of this process since only one point is known.
Within the existing experimental data, much less than for stable nuclei, at the moment we can conclude that there is a clear evidence that for $^{17}\text{F}$ the breakup process gives a rather small contribution to the reaction cross section. Indeed the reaction cross section is very similar to the fusion one (at least in the region where both data are available), the inclusive breakup is nearly one order of magnitude smaller and the exclusive breakup is a further order of magnitude smaller. This behavior is also predicted by the theory [Lia03].

We would like to underline once more that for this system our conclusions are preliminary due to the limited amount of data as well as to the low statistics. Nonetheless we may say that breakup effects seem to be very small for $^{17}\text{F} + ^{208}\text{Pb}$ system, though the very small $^{17}\text{F}$ binding energy might indicate a different behavior. This suggests that parameters, other than binding energy, probably related to the projectile structure, have to taken into account.

![Graph](image.png)

**Figure 6.2:** Experimental cross sections for total reaction and fusion in the system $^{19}\text{F} + ^{208}\text{Pb}$ around the Coulomb barrier. Data labeled “fusion 1” are taken from an experiment performed at China Institute of Atomic Energy (CIAE), Beijing [Lin01] while “fusion 2” data have been collected at ANL [Reh98].

In fact the comparison between the elastic scattering differential cross sections (see Par. 5.3.1 and Fig. 5.7) measured for similar mass systems in the energy range around the Coulomb barrier showed that the radioactive weakly bound $^{17}\text{F}$ nucleus behaves very similarly to the stable well-bound
Oxygen isotopes $^{16,17}\text{O}$, where no breakup excitations are expected. The comparison with the stable Fluorine isotope $^{19}\text{F}$ is more delicate since there is the additional possibility to excite a ground state rotational band and the deduced reaction cross section for this isotope results to be about three times larger than in the $^{17}\text{F}$ case.

Fig. 6.2 presents the fusion [Lin01, Reh98] and the reaction cross sections evaluated from elastic scattering measurements [Lin01] for the system $^{19}\text{F} + ^{208}\text{Pb}$ at Coulomb barrier energies. Since $^{19}\text{F}$ is bound by 4.00 MeV, we do not expect breakup processes, at least in this energy range. The compared analysis of Figs. 6.2 and 6.1 indicates that both nuclei behave in a similar way with no strong processes competing with fusion, as a consequence there is no reduction of the complete fusion cross section. In any case more accurate data for the system $^{17}\text{F} + ^{208}\text{Pb}$ would be desirable.

### 6.2 The system $^{11}\text{Be} + ^{209}\text{Bi}$

Experimental data for the system $^{11}\text{Be} + ^{209}\text{Bi}$ at the Coulomb barrier are available for the following processes:

- **scattering** from 40 MeV to 48 MeV (this work);
- **fusion** for beam energies ranging from 36 MeV to 52 MeV [Sig04].

Fig. 6.3 presents the experimental fusion cross sections and the reaction cross sections deduced from elastic scattering measurements. Unfortunately no information for either the inclusive or the exclusive breakup process $^{11}\text{Be} \rightarrow ^{10}\text{Be} + n$ are presently available. However, a detailed analysis of our experimental data aiming to search for breakup events, should allow to evaluate the breakup cross sections, or at least a lower limit, even with low statistical accuracy.

The fusion cross section data originate from two completely independent measurements performed at RIKEN, which gave sizeable differences between the yields of both fission fragments and $\alpha$-particles emitted from the ground states of the produced evaporation residues. In addition, we cannot exclude a priori that $^{11}\text{Be}$ complete fusion data include a contribution coming from the incomplete fusion of $^{10}\text{Be}$ following an $^{11}\text{Be}$ breakup process, since both fusion processes populate the same evaporation residues.

Fig. 6.3 also reports two theoretical predictions for the total fusion cross section. The first, based on the code CCFULL [Hag99], adopts a collective approach that handles very well the target/projectile inelastic excitations but does not properly consider the projectile breakup channel. The second calculation was performed within the Coupled Discretized Continuum Channels
Figure 6.3: Experimental cross sections for total reaction and fusion in the system $^{11}$Be + $^{209}$Bi at Coulomb barrier energies. Two theoretical predictions for the fusion cross section, one based on the CCFULL collective approach and the other within the CDCC formalism, have also been plotted.

(CDCC) formalism [Aus87]. This approach simultaneously included (i) the effect of the projectile halo structure on the projectile-target interaction potential, (ii) the projectile breakup due to inelastic excitations to partial waves into the continuum induced from the interaction between the projectile fragments and the target and (iii) all continuum couplings (bound-continuum and continuum-continuum). The inelastic excitations of the target, that should have a rather weak influence on the total fusion cross section, and transfer channels are not considered in this approach.

The CCFULL calculations reproduce quite well the trend of the experimental data in the sub-barrier region. On the other hand, the good agreement between CDCC predictions and fusion cross sections values above the Coulomb barrier indicates the relevance of the coupling to the breakup channels in this energy range. Therefore, we may conclude that both approaches are in principle good, since the experimental data do not help us to determine whether the larger influence on the fusion process is provided by single-particle excitations (much better taken into account by the CDCC approach) of collective modes (properly considered by the CCFULL formalism).

However, an accurate theoretical description of the whole interaction should also properly include the incomplete fusion channel, i.e. the possibility that one of the projectile fragments produced in a breakup process
could be captured by the target. This reaction channel could have in principle a fairly large cross section, since Fig. 6.3 shows that for the system $^{11}$Be + $^{209}$Bi the fusion cross section does not exhaust the reaction cross section.

This scenario is very similar to those observed for the systems $^6$Li + $^{208}$Pb [Sig01c], $^6$He + $^{209}$Bi [Agu00] and $^9$Be + $^{209}$Bi [Sig01a] (see Ch. 1). For all these reactions induced by light weakly bound projectiles, strong inclusive breakup channels have been observed at colliding energies around the Coulomb barrier. These processes have cross sections larger than the fusion ones, especially below the barrier. In addition, adding up the inclusive breakup and the fusion cross sections one exhausts the total reaction cross sections deduced from elastic scattering data. This confirms that these two channels should be the strongest ones at Coulomb barrier energies for reaction involving light loosely bound projectiles.

At the light of these considerations, we hypothesize that also for the system $^{11}$Be + $^{209}$Bi the discrepancy between the total reaction cross section and the fusion cross section could arise from a strong $^{11}$Be → $^{10}$Be + $n$ breakup channel. This process might have two components: one with both fragments in the exit channel and the other with only one outgoing fragment, being the other captured by the target. Of course, this hypothesis has to be verified experimentally and a measurement with higher statistical accuracy and better energy resolution is needed to provide a clearer description of the whole interaction.

### 6.3 Reaction cross sections for light RIB’s

We can eventually compare the reaction cross sections for similar mass systems in the region around $^{11}$Be. Fig. 6.4 shows the reaction cross section at the Coulomb barrier for the following systems: $^{11}$Be + $^{209}$Bi ($S_n = 0.504$ MeV), $^9$Be + $^{209}$Bi [Sig00] ($S_n = 1.573$ MeV), $^8$Li + $^{208}$Pb [Kol02] ($S_n = 2.033$ MeV), $^6$Li + $^{208}$Pb [Kee94] ($S_d = 1.475$ MeV) and $^6$He + $^{209}$Bi [Agu00] ($S_n = 0.945$ MeV). Data have been divided by the square of the sum of projectile and target radii $R^2$ and plotted as a function of the ratio $E_{cm}/V_C$ for a better comparison between different reactions. All these systems involve very weakly bound projectiles interacting with high-Z targets. Data with high statistical accuracy are available only for stable nuclei $^6$Li and $^9$Be, which are however the most weakly bound nuclei among all the stable ones.

As we can see, the system $^6$He + $^{209}$Bi shows the highest interaction probability, in particular in the sub-barrier region, where the larger contribution arises from the inclusive breakup channel $^6$He → $^4$He ($+ n + n$). Also the reactions induced by the radioactive nuclei $^8$Li and $^{11}$Be have very large re-
Figure 6.4: Reaction cross sections deduced from experimental elastic scattering measurements for five similar mass systems in the energy range around the Coulomb barrier. Data are been divided by $R^2$, with $R$ sum of the projectile and target radii, and plotted versus $E_{cm}/V_C$, with $V_C$ Coulomb barrier, for a better comparison among all different nuclei. The following $V_C$ values were adopted: 37.7 MeV (38.4 MeV) for the system $^{11}\text{Be} + ^{209}\text{Bi}$, 28.8 MeV (29.5 MeV) for $^8\text{Li} + ^{208}\text{Pb}$ and 18.6 MeV for $^6\text{He} + ^{209}\text{Bi}$.

Action cross section at Coulomb barrier energies. Their absolute values at the same ratio $E_{cm}/V_C$ are smaller than for $^6\text{He}$, even if these nuclei are less bound. In addition, although the binding energies of $^8\text{Li}$ and $^{11}\text{Be}$ differ by a factor 4, the two reaction cross sections are similar.

The reaction cross sections for the systems $^6\text{Li} + ^{208}\text{Pb}$ and $^9\text{Be} + ^{209}\text{Bi}$ resulted to be smaller than for the previous three reactions and rather similar to each other. This is reasonable since these nuclei are more tight and they have almost the same binding energy.

An overall comparison of these reactions induced by light weakly bound projectiles shows that in the energy range around the Coulomb barrier, the reaction cross section increases moving from stable isotopes to radioactive ones. This increase does not seem to depend only on the projectile binding energy. Other factors, most likely connected to the specific nuclear structure could influence the total reaction cross section, such as, for example, cluster structure, population of possible rotational band, single particle excitations, ... However, this trend could not be extrapolated to heavier regions, as already observed in this work for Fluorine isotopes, where around the barrier
the reaction cross section for the stable well-bound $^{19}\text{F}$ resulted to be three times larger than for the weakly bound radioactive $^{17}\text{F}$. 
Chapter 7

Conclusions

In the present work the scattering processes of $^{17}$F nuclei from a $^{208}$Pb target and of $^{11}$Be ions from $^{209}$Bi have been studied in the energy range around the Coulomb barrier. The light charged particles produced in the reactions were detected using the EXODET apparatus. The large solid angle coverage and high position accuracy of this array makes it especially well suited to study nuclear reactions induced by low intensity and poor energy and position resolution RIB’s.

The $^{17}$F secondary beam was produced through the inverse kinematics reaction $p(^{17}O,^{17}F)n$ at the Argonne National Laboratory (USA). The $^{17}$F scattering differential cross section at 90.4-MeV beam energy was measured in the angular range from $115^\circ \leq \theta \leq 155^\circ$. The collected data were analyzed within the optical model framework in order to get the best-fit potential parameters, to calculate the real and the imaginary strong absorption radii, to define the potential around these radial points and to extract the reaction cross section at Coulomb barrier energies. A similar analysis has also been performed for the scattering data collected for the same system at 98-MeV beam energy by part of our collaboration.

The results obtained at 90.4 and 98 MeV were compared with those evaluated for the system $^{19}$F + $^{208}$Pb at about the same $E_{cm}/V_{c}$-values. Contrary to $^{19}$F, the $^{17}$F imaginary strong absorption radius resulted to be systematically $\sim 10\%$ larger than the real one. This suggests that the $^{17}$F imaginary part of the nuclear potential is extended well outside the nucleus core. We believe that one of the origin of this effect is the very low binding energy of the last proton.

The elastic scattering angular distributions of these two Fluorine isotopes have been compared with those obtained for the similar mass systems $^{16,17}$O + $^{208}$Pb. The $^{17}$F behavior is much more similar to that of the Oxygen isotopes rather than to the $^{19}$F one. This fact was quite surprising, since the
radioactive and weakly bound $^{17}\text{F}$ was expected to behave differently from the well-bound $^{16}\text{O}$ and $^{17}\text{O}$ nuclei. As a consequence, the reaction cross section for system $^{19}\text{F} + ^{208}\text{Pb}$ turns out to be a factor 3 larger than for the reactions $^{17}\text{F}, ^{16}, ^{17}\text{O} + ^{208}\text{Pb}$. From this observation we conclude that at Coulomb barrier energies the probability to excite collective modes, even in a well-bound nucleus like $^{19}\text{F}$, is larger than the breakup probability in $^{17}\text{F}$.

We have also performed a first direct measurement of the breakup process $^{17}\text{F} \rightarrow ^{16}\text{O} + p$ cross section below the Coulomb barrier. The average value at backward angles ($\theta = 125^\circ$) for the differential cross section was $(2.6 \pm 1.2)$ mb/sr. If this value is confirmed also at forward angles, the cross section of the exclusive (both $^{16}\text{O}$ and $p$ fragments in the exit channel) breakup process results to be rather small. Similar results for this system have already been obtained at 170-MeV bombarding energy, well above the breakup threshold, where the cross section for the exclusive breakup [Lia00] was found to be much smaller than the inclusive breakup [Lia02] (only $^{16}\text{O}$ in the exit channel being the other fragment captured by the target). Therefore the data support the hypothesis that the at Coulomb barrier energies the strongest breakup channel has only one outgoing fragment ($^{16}\text{O}$), similarly to what observed for the same system at higher energy [Lia02] and for the system $^6\text{Li} + ^{208}\text{Pb}$ in the energy range around the Coulomb barrier [Sig03].

An overall comparison between the cross sections of all measured processes (fusion, inclusive and exclusive breakup) and the reaction cross section deduced from the elastic scattering shows that for the system $^{17}\text{F} + ^{208}\text{Pb}$ at Coulomb barrier energies there is no strong process competing with fusion.

In the second experiment, the $^{11}\text{Be}$ secondary beam was produced at RIKEN (Japan) via In-Flight fragmentation of a high energy $^{13}\text{C}$ primary beam on a thick Beryllium target. The selected ions were separated from all the other projectile fragments via the RIPS spectrometer and their energy was then slowed down to the required energy range by using an Aluminium degrader. The resulting secondary beam had a large energy spread and the data analysis was carried out dividing the collected data in 2-MeV bins.

The $^{11}\text{Be}$ scattering differential cross sections have been measured from 40- to 48-MeV beam energy. Data have been grouped into $10^\circ$-bins; two points have been evaluated at forward angles and four in the backward direction for each energy bin. An optical model best-fit analysis of the scattering angular distributions have been performed to get the consistent potential parameters and to deduce the reaction cross sections in the energy range around the Coulomb barrier.

The comparison between the systems $^{11}\text{Be} + ^{209}\text{Bi}$ and $^9\text{Be} + ^{209}\text{Bi}$ shows a similar behavior for the scattering differential cross sections, while the reaction cross sections resulting from the optical model analysis turn out to be
larger for the radioactive isotope. Therefore, within the accuracy of the collected data, this observation suggests that, differently from what previously observed for the Fluorine isotopes, the probability to excite a ground state rotational band in the $^9\text{Be}$ nucleus only partly compensates the effects due to the $^{11}\text{Be}$ halo structure and much lower binding energy.

We also observed that for the system $^{11}\text{Be} + ^{209}\text{Bi}$ the fusion cross section does not exhaust the reaction cross section deduced from the scattering data analyzed in this work. This scenario is completely different from those drawn for the $^{17}\text{F}$ and $^{19}\text{F}$, where the fusion process was the strongest reaction channel at Coulomb barrier energies. The $^{11}\text{Be}$ behavior was found to be much more similar to those observed for the systems $^6\text{Li} + ^{208}\text{Pb}$ [Sig01c], $^6\text{He} + ^{209}\text{Bi}$ [Agu00] and $^9\text{Be} + ^{209}\text{Bi}$ [Sig01a]. In all these reactions an inclusive breakup channel with a fairly large cross section (especially in the sub-barrier region) has been measured.

For these reasons, we hypothesize that the difference between the reaction cross section and the fusion cross section in the system $^{11}\text{Be} + ^{209}\text{Bi}$ could be due to a strong $^{11}\text{Be} \to ^{10}\text{Be} + n$ breakup channel. In analogy with the previously mentioned systems, this process could have two components: one with both fragments in the exit channel and the other with only one outgoing fragment, being the other captured by the target. Of course, all these hypotheses need to be verified by the experiments.

Finally, we may underline that the reported data suffer of the low intensity and poor energy resolution and emittance of the presently available RIB's. Therefore measurements with higher statistical accuracy and better would be desirable in order to get a deeper understanding of all the reaction mechanisms. However present data represent the nowadays limit for experiments with first generation RIB's and give a first important insight on the nuclear structure parameters involved in this kind of reaction.
Appendix A

Theoretical analysis with Fresco

A brief overview of the theoretical analysis performed with the code Fresco [Tho88] will be presented in this Chapter. We will first describe the formalism of the Distorted Wave Born Approximation (DWBA) used to calculate the cross sections for both elastic scattering process and inelastic excitations of the projectile and the target in the systems $^{17}$F + $^{208}$Pb and $^{11}$Be + $^{209}$Bi. Then we will show an example of Fresco input card and an input card of the subroutine SFresco used to minimize the $\chi^2$ between the experimental data and the theoretical predictions.

A.1 The Distorted Wave Born Approximation

The DWBA formalism is a more sophisticated version of the Born Approximation (BA). Therefore we will first present the Born Approximation, since it is a useful starting point to describe the DWBA formalism. In this paragraph, we basically report the formalism used in [Sat80].

If we have a particle with mass $m$ in a time-independent potential $V(\vec{r})$, the particle wave-function $\chi(\vec{r})$ is the solution of the Schrödinger equation:

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) \right] \chi(\vec{r}) = H \chi(\vec{r}) = E \chi(\vec{r}) \quad (A.1)$$

Equation A.1 is conveniently rewritten as:

$$\left[ \nabla^2 - U(\vec{r}) + k^2 \right] \chi(\vec{r}) = 0 \quad (A.2)$$
where the wave number $k = \sqrt{2mE/\hbar^2}$ and $U = 2mV/\hbar^2$. Scattering theory predicts that the solution of A.2 could be written as the sum of an “incident” plane wave (with wave number $\vec{k}$) and an outgoing “scattered” wave:

$$\chi(\vec{k}, \vec{r}) = e^{i\vec{k}\cdot\vec{r}} - \frac{1}{4\pi} \int \frac{e^{i\vec{k}\cdot\vec{r}'}}{|\vec{r} - \vec{r}'|} U(\vec{r}') \chi(\vec{k}, \vec{r}') d\vec{r}'$$  \hspace{1cm} (A.3)

Let us consider the asymptotic behavior ($r \rightarrow \infty$) of the wave function. Under the hypothesis that $U(\vec{r}')$ is a short-range interaction, we can write:

$$\chi(\vec{k}, \vec{r}) \overset{r \rightarrow \infty}{\rightarrow} e^{i\vec{k}\cdot\vec{r}} - \frac{e^{ikr}}{4\pi r} \int e^{-i\vec{k}\cdot\vec{r}'} U(\vec{r}') \chi(\vec{k}, \vec{r}') d\vec{r}'$$  \hspace{1cm} (A.4)

where the vector $\vec{r}'$ has the magnitude of $\vec{k}$ and the direction of $\vec{r}'$. If we introduce the scattering amplitude:

$$f(\theta, \phi) = -\frac{1}{4\pi} \int e^{-i\vec{k}\cdot\vec{r}'} U(\vec{r}') \chi(\vec{k}, \vec{r}') d\vec{r}'$$  \hspace{1cm} (A.5)

Equation A.4 becomes:

$$\chi(\vec{k}, \vec{r}) \overset{r \rightarrow \infty}{\rightarrow} e^{i\vec{k}\cdot\vec{r}} + f(\theta, \phi) \frac{e^{ikr}}{r}$$  \hspace{1cm} (A.6)

In case of a “pure” elastic scattering process, the relationship between $f(\theta, \phi)$ and the differential cross section is given by the well-known formula:

$$\frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2$$  \hspace{1cm} (A.7)

where $\theta$ e $\phi$ are the polar angles individuated by the vector $\vec{k}'$.

The integral equation A.4, asymptotic solution of the Schrödinger equation A.2, is the starting point of any approximation of the scattering theory.

For example, if we assume that the nuclear interaction is weak compared to the kinetic energy, the amplitude of the outgoing wave results to be rather small and consequently the wave function $\chi(\vec{k}, \vec{r})$ can be described in a first approximation with a plane wave $e^{i\vec{k}\cdot\vec{r}}$. This is the Born Approximation. In this case the scattering amplitude turns out to be:

$$f_{BA}(\theta, \phi) = -\frac{1}{4\pi} \int e^{-i\vec{k}\cdot\vec{r}'} U(\vec{r}') e^{i\vec{k}\cdot\vec{r}'} d\vec{r}'$$  \hspace{1cm} (A.8)

Equation A.8 can be rewritten as:

$$f_{BA}(\theta, \phi) = -\frac{1}{4\pi} \int e^{-i\vec{q}\cdot\vec{r}'} U(\vec{r}') d\vec{r}'$$  \hspace{1cm} (A.9)
by introducing the vector \( \vec{q} = \vec{k} - \vec{k}' \), describing the momentum transferred to the target. The Born Approximation is not very useful in nuclear physics, since the nucleon kinetic energy and the nuclear potential are usually comparable. For this reason we use a more sophisticated approach: the DWBA formalism. Let us suppose that the potential \( U \) can be written as the sum of two terms, \( U = U_1 + U_2 \), and that we know the solution of the following equation:

\[
\left[ \nabla^2 - U_1(\vec{r}) + k^2 \right] \chi_1(\vec{r}) = 0 \tag{A.10}
\]

This equation has two kinds of solution: the first is the sum of an incoming plane wave and an outgoing “scattered” wave function \( \chi_1^{(+)}(\vec{k}, \vec{r}) \) and the second consists of an outgoing plane wave and an ingoing “scattered” wave function \( \chi_1^{(-)}(\vec{k}, \vec{r}) \). The relationship between the two scattered waves is:

\[
\chi_1^{(-)}(\vec{k}, \vec{r}) = \chi_1^{(+)}(\vec{k}, \vec{r})^* \tag{A.11}
\]

Within this formalism we can see that the asymptotic solution \( \chi(\vec{k}, \vec{r}) \) of equation A.2 results to be:

\[
\chi(\vec{k}, \vec{r}) \underset{r \to \infty}{=} \chi_1^{(+)}(\vec{k}, \vec{r}) - \frac{e^{ikr}}{4\pi r} \int \chi_1^{(-)}(\vec{k}', \vec{r'}) U_2(\vec{r'}) \chi(\vec{k}, \vec{r'}) d\vec{r'} \tag{A.12}
\]

The full scattering amplitude consists of two contributions: the first one arising from the \( \chi_1^{(+)}(\vec{k}, \vec{r}) \) and the other from the second term of A.12:

\[
f(\theta, \phi) = f_1(\theta, \phi) - \frac{1}{4\pi} \int \chi_1^{(-)}(\vec{k}', \vec{r'}) U_2(\vec{r'}) \chi(\vec{k}, \vec{r'}) d\vec{r'} \tag{A.13}
\]

where \( f_1(\theta, \phi) \) can be derived from A.5 with \( U_1 \) and \( \chi_1 \) instead of \( U \) and \( \chi \), respectively.

The problem is only formally solved, since A.12 is still an integral equation containing the exact (unknown) solution \( \chi(\vec{k}, \vec{r'}) \). In analogy with the previous case, we can approximate \( \chi \) with \( \chi_1 \), solution of Eq. A.10 for the potential \( U_1 \). The replacement will result good if \( U_2 \) is weak in comparison with \( U_1 \). Within this approximation the scattering amplitude results:

\[
f_{DWBA}(\theta, \phi) = f_1(\theta, \phi) - \frac{1}{4\pi} \int \chi_1^{(-)}(\vec{k}', \vec{r'}) U_2(\vec{r'}) \chi_1^{(+)}(\vec{k}, \vec{r'}) d\vec{r'} \tag{A.14}
\]

In case of inelastic excitations and/or nucleon transfer reactions, \( U_1 \) is generally chosen to describe the elastic scattering process and \( U_2 \) for the inelastic transitions. The validity of the DWBA formalism will result rather good if the inelastic processes can be treated as perturbations of the pure elastic scattering process.
A.2 FRESCO code

We present in this Section an example of the theoretical analysis performed with the code FRESCO within the DWBA formalism. In particular we will see how we calculated the differential cross sections for the elastic scattering process and for the inelastic excitations to the first $^{17}$F state at 0.4953 MeV and to the first $^{208}$Pb level at 2.614 MeV in the system $^{17}$F + $^{208}$Pb at 90.4 MeV.

A.2.1 FRESCO input card

Card 0: heading.

09/07/2003 17F+208Pb elastic & 2 inelastic scatt 90.4 MeV

Card 1: radial definition of the partial waves.

0.03 20. 0.25 0.025 1.5 -0.00

Card 2: partial wave coupling.

0. 150. -.001 F 0 1 0 0.

Card 3: angular range ($\theta = 20-180^\circ$, with $2^\circ$-bins) of the calculation.

1 20.0 180. 2.00

Card 4: formalism (DWBA or Coupled Channel) choice.

-0.001 11000 3 0 24 0 0 8

Card 5: outputs definition.

3 0 0 0 1 1 0 0 0 0

Card 6: reaction ($^{17}$F + $^{208}$Pb).

17F 17.0021 9.0 3 208Pb 207.977 82.0 +0.0000

Cards 7: projectile and target levels.

2.5 +1 0.0 1 0.0 +1 0.0 F F

0.5 +1 0.4953 1 1

1 1 3.0 -1 2.614

Card 8: blank card.
Card 9: Coulomb potential.

1 0 208.0 17.00 1.20

Card 11: deformations of the Coulomb potential due to the target/projectiles excitations taken into account.

112 1.00 11.4 780.95
   1 1 2 0.00
   2 1 2 11.4
   1 2 2 11.4
   -2 2 1 0.00
113 1.00 11.4 780.95
   3 1 3 780.95
   1 3 3 780.95
   -3 3 2 0.00

Card 10: nuclear potential.

1 1 60.55 1.24 0.53 5.71 1.24 0.53

Card 11: deformations of the nuclear potential due to the considered inelastic excitations of the target and projectile.

112 101.00 1.72 0.79
   1 1 2 0.00
   2 1 2 1.72
   1 2 2 1.72
   -2 2 1 0.00
113 101.00 1.72 0.79
   3 1 3 0.79
   1 3 3 0.79
   -3 3 2 0.00

Card 12: blank card

0

Cards 13-18: blank cards since no transfer reactions were considered and no spectroscopic factors were needed for this calculation.

0
0

Card 19: energy in the laboratory frame.

90.4
A.3 Search version SFRESCO

SFRESCO is a program which searches for a $\chi^2$ minimum when comparing the outputs of FRESCO with experimental sets of data, using the MINUIT [Min98] search routines.

A.3.1 SFRESCO search file

The inputs of SFRESCO specify the FRESCO input and output files, the number and types of search variables, and the experimental data sets.

Card 1: input file, output file, number of variables and number of experimental data sets.

'mf73.in' 'mf73.out' 6 1

Cards 2, &variable: kind of the variable (potential variable), name of variable (depth $V$, $W$, radius $r_0$, $r_w$ or diffuseness $a$, $a_w$ for the real and the imaginary potential), fixed value of the parameter (e.g. potential=1.20) or number of the FRESCO input card with the potential values (e.g. kp=1 pline=2 col=1).

&variable kind=1 name='V' kp=1 pline=2 col=1 / 
&variable kind=1 name='r0' potential=1.20 / 
&variable kind=1 name='a' potential=0.48 / 
&variable kind=1 name='W' kp=1 pline=2 col=4 / 
&variable kind=1 name='rw' potential=1.20 / 
&variable kind=1 name='aw' potential=0.48 /

Cards 3, &data: type of data set (e.g. type=0 for angular distribution at fixed energy), dimension (e.g. iscale=-1 for dimensionless and idir=1 for Ratio to Rutherford), laboratory (lab=T) or center-of-mass (lab=F) frame, errors (e.g. abserr=T for absolute errors).

&data type=0 iscale=-1 idir=1 lab=F abserr=T/

Data input.

118.29 1.01 0.13
119.25 1.00 0.09
120.22 0.97 0.07
...
154.20 0.69 0.10
155.13 0.62 0.12
156.05 0.59 0.19
&
A.3.2 SFresco input

The inputs of SFresco consist of two sets of cards:
Card 1: name of the SFresco search file.

mfs73.in

Card 2: call MINUIT.

min

Card 2: fixed variables.

fix 2
fix 3
fix 5
fix 6

Card 2: MINUIT command.

migrad

Card 2: end of MINUIT.

end

Card 2: query status.

q

Card 2: list of datasets with relative $\chi^2$.

show

Card 2: plot of the results.

plot
Appendix B

Monte-Carlo simulation

In this Appendix we present the source file used to estimate the geometrical efficiency of the EXODET apparatus via Monte-Carlo simulation. We have to underline that the results do not depend on the target/projectile choice and on beam energy, however in the present case we refer to the elastic scattering process of $^{11}\text{Be}$ nuclei from a $^{209}\text{Bi}$ target at a fixed externally settable beam energy $E_0$. The FWHM of the beam at the target position was set to 1.0 cm. The code has been developed in the Linux environment with the C/C++ language.

B.1 Monte-Carlo code

Title and comments

```
//            ***        EXODET_RIBs1.C       ***
//
// This macro calculates the geometrical efficiency of the EXODET
// apparatus for the detection of $^{11}\text{Be}$ elastic scattering events
// from a $^{209}\text{Bi}$ target. The output consists of 16 matrices with the
// solid angles (in msr) covered by each polar angle theta (with
// 1-degree bin) inside any EXODET DeltaE strip. The first 8 matrices
// consider only the DeltaE strip-structure while the other 8
// take into account the whole EXODET pixel-structure.
// Input parameters: event number (imax)
//                 beam energy (E0)
```

Include files

```
#include <time.h>       // define time() 
```

143
#include "randomc.h"     // classes for random number generators
#include <iostream>
#include "mersenne.cpp"   // members of class TRandomMersenne
#include <math.h>
#include "exodet.h"       // definition of the EXODET functions
using namespace std;

/* Be elastic scattering parameters definition */
#define M_BE       10266.69619
#define M_BI       194665.6449
#define PI         3.1415926535

int main()
{
    long int imax;
    float E0, Ex = 0, E_Be, theta_Be = 0, phi_Be= 0;
    float energy_spread, risoluzione, spot,
       offset_xt, offset_yt, spot_ratio;
    float V, Vx, Vy, Vz, tx = 0, ty = 0;
    unsigned tel;
    float R_de, R_er, offset_backward, offset_forward;
    float offset_left, offset_top, offset_right, offset_bottom;
    float X_de, Y_de, Z_de, X_er, Y_er, Z_er, X_T = 0, Y_T = 0;
    offset_backward = 0.4;
    offset_forward = 0.4;
    spot = 0.425;       // spot = 0.425 corresponds to FWHM = 1.0 cm
    offset_xt = 0.0;    // test value
    offset_yt = 0.0;    // test value
    spot_ratio = 1.0;   // test value
    R_de = 2.625;       // half edge of the DeltaE boxes
    R_er = 3.145;       // half edge of the E boxes
    offset_left = -2.375;
    offset_top = -2.625;
    offset_right = -2.625;
    offset_bottom = -2.375;

    // Definition and initialization of the output vectors
    unsigned long angolosolido_de[8][100][180],
               angolosolido_er[8][100][180];
unsigned long scattering_1[8], scattering_2[8], scattering_3[8], scattering_4[8], scattering_5[8];
unsigned long rearside_2[8], rearside_3[8],
        rearside_4[8], rearside_5[8];

for(int i = 0; i < 8; i++)
{
    scattering_1[i] = 0; scattering_2[i] = 0; scattering_3[i] = 0;
    scattering_4[i] = 0; scattering_5[i] = 0; rearside_2[i] = 0;
    rearside_3[i] = 0; rearside_4[i] = 0; rearside_5[i] = 0;
}

for (unsigned int t = 0 ; t < 8; t++)
{
    for (unsigned int s = 0 ; s < 100; s++)
    {
        for (unsigned int r = 0 ; r < 180; r++)
        {
            angolosolido_de[t][s][r] = 0;
            angolosolido_er[t][s][r] = 0;
        }
    }
}

Input data

    fprintf(stderr,"event number...\n");
    cin >> imax;
    fprintf(stderr,"beam energy (MeV)\n");
    cin >> E0;

Change of the offset for the backward hemisphere, since this parameter is
initially inserted with its absolute value.

    offset_backward = - offset_backward - 5.0;

Event generation

    //beginning of the for over the events

    for(long i = 1 ; i <= imax; i++)
    {

long int seed = time(0) + i * 127;

TRandomMersenne rg(seed);

Target coordinates

The target coordinates \((x_T, y_T)\) hit by the incident particles have been generated assuming a 2-dimensional Gaussian beam distribution along the \(x\) and \(y\) axes at the target plane \((z_T = 0)\):

\[
P_x(x) dx = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} dx \tag{B.1}
\]

\[
P_y(y) dy = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy \tag{B.2}
\]

These distributions can be written as a unique function depending on the radial coordinate \(r = \sqrt{x^2 + y^2}\) and the polar angle \(\theta\) between the radial direction and the \(x\) axis:

\[
P_{r,\theta}(r, \theta) dr d\theta = \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}} r dr d\theta \tag{B.3}
\]

As we can see, Equation B.3 separately depends on the parameters \(r\) and \(\theta\). Therefore we can use two separate distributions, \(P_r(r)\) and \(P_\theta(\theta)\), to describe each dependence:

\[
P_r(r) dr = \frac{1}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} r dr \tag{B.4}
\]

\[
P_\theta(\theta) d\theta = \frac{1}{2\pi} d\theta \tag{B.5}
\]

One can easily see that the probability distributions \(P_r(r)\) and \(P_\theta(\theta)\) can be generated with two random numbers \(\eta, \xi \in [0,1]\) by means of the following formulas:

\[
r = \sigma \sqrt{2\ln \left( \frac{1}{1 - \eta} \right)} \tag{B.6}
\]

\[
\theta = 2\pi \xi \tag{B.7}
\]

In our Monte-Carlo simulation we adopted this random generation of the target coordinates, since it corresponds to the most efficient generation of a 2-dimensional Gaussian distribution.

    // introduction of the target hit coordinates
    // (Gaussian distribution)
newspot:
    float R_T = spot * sqrt( 2 * log( 1 / (1 - rg.Random() )));
    float Theta_T = 2 * PI * rg.Random();
    X_T = offset_xt + ( R_T * sin(Theta_T));
    Y_T = offset_yt + ( spot_ratio * R_T * cos(Theta_T));

We also imposed the condition that the generated target coordinates fell
with the finite dimension of a “real” target. In fact the Gaussian distribu-
tion covers all the real axis (\([-\infty, +\infty]\]), even if the tails of the distribution
(deviations larger than 3\(\sigma\) from the central value) have very small popu-
lations. In this case we assumed a target radius of 2.0 cm.

    // introduction of the target ‘finite’ dimension
    if ( (pow(X_T,2) + pow(Y_T,2)) > 4.0 )
        goto newspot;

Elastic scattering process

The EXODET geometrical efficiency for a \(^{11}\)Be scattering process from a
\(^{209}\)Bi target was evaluated generating a uniform event distribution over \(4\pi\)
sr. Using the spherical coordinates \(\theta\) (polar angle) and \(\phi\) (azimuthal angle),
this distribution probability is described by the following formula:

\[
P_{\theta,\phi}(\theta, \phi)d\theta d\phi = \frac{1}{4\pi} \sin \theta d\theta d\phi
\]  

(B.8)

Equation B.8 can be separated into two factors, as follows:

\[
P_{\theta}(\theta)d\theta = \frac{1}{2\pi} \sin \theta d\theta
\]  

(B.9)

\[
P_{\phi}(\phi)d\phi = \frac{1}{2\pi} d\phi
\]  

(B.10)

Also in this case, one can easily see that the \(P_{\theta}(\theta)\) and \(P_{\phi}(\phi)\) probability
distributions can be generated by the random numbers \(\mu, \nu \in [0,1]\) using the following formulas:

\[
\theta = \arccos(1 - 2\mu)
\]  

(B.11)

\[
\phi = 2\pi \nu
\]  

(B.12)

We also assigned a scattering energy to each event, according to its scattering
angle \(\theta\). In the present simulation this assignment is not strictly necessary,
since the evaluation of the geometrical efficiency does not depend on the
particle incoming or scattering energy. However it could turn out to be
useful for future developments of the code (evaluation of the energy lost into the target or into the $\Delta E$ layer, ...). The $^{11}$Be scattering energy has been calculated as follows [Mar68]:

$$E(\theta) = E_0 \left( \frac{m_{Be}}{m_{Be} + m_{Bi}} \right)^2 \left( \cos \theta + \sqrt{\left( \frac{m_{Bi}}{m_{Be}} \right)^2 - \sin^2 \theta} \right)^2$$  \hfill (B.13)

where $E_0$ is the beam energy, $m_{Be}$ the $^{11}$Be mass and $m_{Bi}$ $^{209}$Bi mass.

// 11Be scattering angles (theta_Be e phi_Be)

theta_Be = acos (1 - 2 * rg.Random());
phi_Be  = 2 * PI * rg.Random();

// 11Be elastic scattering energy

E_Be = E_0 * pow( M_BE / (M_BE + M_BI), 2) *
      pow( cos(theta_Be) + sqrt((M_BI/M_BE)*(M_BI/M_BE) -
        pow( sin(theta_Be), 2)), 2);

Hit telescope selection

The selection of the EXODET telescope hit by the generated events is performed calculating the $^{11}$Be velocity components along the axes $x, y, z$ of our reference frame and minimizing the flight time between the target hit position and the four forward telescopes (if $\theta \leq 90^\circ$) or otherwise the four telescopes at backward angles. Forward (backward) telescopes have been labeled as follow: tel = 1(5), 2(6), 3(7) and 4(8) for that on the left, top, right and bottom, respectively. The telescope selection was performed using the telescopio subroutine, described in the header file “exodet.h”.

// 11Be velocity (in v/c)

V = sqrt( 2 * E_Be / M_BE);

// 11Be velocity components (in v/c)

Vx = V * sin(theta_Be) * cos(phi_Be);
Vy = V * sin(theta_Be) * sin(phi_Be);
Vz = V * cos(theta_Be);
// selection of the EXODET telescope hit by the scattered 11Be

if (Vz > 0)
{
    tel = telescocio( tx, ty, Vx, Vy, X_T, Y_T, R_de);
}

if (Vz < 0)
{
    tel = 4 + telescocio( tx, ty, Vx, Vy, X_T, Y_T, R_de);
}

// update of the output vector scattering_1[8], describing
// the event number of each subspace individuated by the
// eight EXODET telescopes (check parameter of the random
// generation of the events)

++scattering_1[tel -1];

EXODET $\Delta E$ (DE) and $E$ (ER) coordinates

// calculation of the EXODET DE ($X_{de}$, $Y_{de}$ e $Z_{de}$) and
// ER ($X_{er}$, $Y_{er}$ e $Z_{er}$) coordinates hit by the 11Be
// scattered particle

if (tel == 1 || tel == 5)
{
    X_de = R_de;
    Y_de = tx * Vy + Y_T;
    Z_de = tx * Vz;
    X_er = R_er;
    Y_er = ((R_er - X_T) / Vx) * Vy + Y_T;
    Z_er = ((R_er - X_T) / Vx) * Vz;
}

if ( tel == 3 || tel == 7)
{
    X_de = - R_de;
Y\_de = tx \* Vy + Y\_T;
Z\_de = tx \* Vz;
X\_er = - R\_er;
Y\_er = ((- R\_er - X\_T) \/ Vx) \* Vy + Y\_T;
Z\_er = ((- R\_er - X\_T) \/ Vx) \* Vz;
}

if ( tel == 2 || tel == 6)
{
X\_de = ty \* Vx + X\_T;
Y\_de = R\_de;
Z\_de = ty \* Vz;
X\_er = ((R\_er - Y\_T) \/ Vy) \* Vx + X\_T;
Y\_er = R\_er;
Z\_er = ((R\_er - Y\_T) \/ Vy) \* Vz;
}

if ( tel == 4 || tel == 8)
{
X\_de = ty \* Vx + X\_T;
Y\_de = - R\_de;
Z\_de = ty \* Vz;
X\_er = ((- R\_er - Y\_T) \/ Vy) \* Vx + X\_T;
Y\_er = - R\_er;
Z\_er = ((- R\_er - Y\_T) \/ Vy) \* Vz;
}

**EXODET detection efficiencies**

In the first part of the code we defined 9 output vectors (scattering\_1-5 and rearside\_2-5) to evaluate the geometrical efficiencies of the segmented front sides and continuous rear sides of both $\Delta E$ and $E$ EXODET detectors on their own and also that of the $\Delta E - E$ EXODET telescopes:

- **scattering\_1**: check output vector to control the correct random generation of the scattering events;

- **scattering\_2** and **rearside\_2**: geometrical efficiency of the continuous rear side of the $\Delta E$ EXODET detectors;

- **scattering\_3**: geometrical efficiency of the segmented front side of the $\Delta E$ EXODET detectors ($\Delta E$ strip-structure);
• **rearside\_3**: geometrical efficiency of the $E$ EXODET detector rear sides;

• **rearside\_4**: geometrical efficiency of the $E$ EXODET detector segmented front sides ($E$ strip-structure);

• **rearside\_5**: geometrical efficiency of the continuous detector rear sides of the EXODET telescopes;

• **scattering\_4**: geometrical efficiency of $\Delta E$ strip-structure combined to the geometrical efficiency of the $E$ detector continuous rear sides;

• **scattering\_5**: geometrical efficiency of the segmented detector front sides of the EXODET telescopes (EXODET pixel-structure).

The used subroutines `scattering`, `rearside1` and `rearside2` are listed in the header file "exodet.h".

```c
// selection of the events detected by the EXODET telescopes
// with different conditions imposed by the geometry of
// the detectors and the inter-strip dead layers.

switch(tel)
{
    case 1:
        scattering( Z_de, Y_de, offset_forward, offset_left,
                   Z_er, Y_er, scattering_2[tel - 1], scattering_3[tel - 1],
                   scattering_4[tel - 1], scattering_5[tel - 1]);

        rearside1( Z_de, Y_de, offset_forward, offset_left,
                   Z_er, Y_er, rearside_2[tel - 1], rearside_5[tel - 1]);

        rearside2( Z_de, Y_de, offset_forward, offset_left,
                   Z_er, Y_er, rearside_3[tel - 1], rearside_4[tel - 1]);
        break;

    case 2:
        scattering( Z_de, X_de, offset_forward, offset_top,
                   Z_er, X_er, scattering_2[tel - 1], scattering_3[tel - 1],
                   scattering_4[tel - 1], scattering_5[tel - 1]);

        rearside1( Z_de, X_de, offset_forward, offset_top,
                   Z_er, X_er, rearside_2[tel - 1], rearside_5[tel - 1]);
```

rearsid2(Z_de, X_de, offset_forward, offset_top, 
Z_er, X_er, rearsid3[tel - 1], rearsid4[tel - 1]); 
break;

The case tel = 3 (4) can be easily obtained from the case tel = 1 (2) replacing offset_left (offset_top) with offset_right (offset_bottom).

case 5:
  scattering(Z_de, Y_de, offset_backward, offset_left, 
  Z_er, Y_er, scattering2[tel - 1], scattering3[tel - 1], 
  scattering4[tel - 1], scattering5[tel - 1]);

case 6:
  scattering(Z_de, X_de, offset_backward, offset_left, 
  Z_er, X_er, rearsid2[tel - 1], rearsid5[tel - 1]);
break;

case 5:
  scattering(Z_de, Y_de, offset_backward, offset_left, 
  Z_er, Y_er, scattering2[tel - 1], scattering3[tel - 1], 
  scattering4[tel - 1], scattering5[tel - 1]);

case 6:
  scattering(Z_de, X_de, offset_backward, offset_left, 
  Z_er, X_er, rearsid2[tel - 1], rearsid5[tel - 1]);
break;

} 

Similarly to the forward telescopes, the case tel = 7 (8) can be easily obtained from that tel = 5 (6) replacing offset_left (offset_top) with offset_right (offset_bottom).

Evaluation of the polar and solid angles covered by the $\Delta E$ strips

In the last part of the code, we calculate the range of polar angles $\theta$ subtended by each $\Delta E$ strip and also the solid angle pertinent to each $\theta$ angle inside any $\Delta E$ strip. Two evaluations have been performed: the first
one (angolosolido_de[tel - 1][jz][j]) considering only the $\Delta E$ strip-
structure, (angolosolido_er[tel - 1][jz][j]) considering the whole $\Delta E -$
$E$ EXODET pixel-structure,

```c
switch(tel)
{
    case 1:
        for (int j = 0; j < 90; j++)
        {
            if ( (theta_Be >= (0.008727 + j * 0.017453)) &&
                 (theta_Be <= (0.008727 + (j + 1) * 0.017453)) )
            {
                for (int jz = 0; jz < 100; jz++)
                {
                    if( ( ((Z_de >= (offset_forward + 0.0025 +
                            jz * 0.0500)) && (Z_de <= (offset_forward -
                            0.0025 + (jz+1) * 0.0500))) && (Y_de >=
                            offset_left)) && (Y_de <= (offset_left + 5.0)))
                    {
                        ++angolosolido_de[0][jz][j];
                        for (int jxy = 0; jxy < 100; jxy++)
                        {
                            if((( (Z_er >= offset_forward) && (Z_er <=
                                  (offset_forward + 5.0)) ) && (Y_er >=
                                  (offset_left + 0.0025 + jxy *0.0500) )) &&
                                  (Y_er <= (offset_left - 0.0025
                                      + (jxy+1) * 0.0500)))
                            {
                                ++angolosolido_er[0][jz][j];
                            }
                        }
                    }
                }
            }
        }
    
    break;
}
```

The cases tel = 2, 3, 4 can be easily obtained from that tel = 1 through opportune variations of the parameters Y_de, Y_er and offset_left and are not presented in this Appendix.

```
case 5:
```
for (int j = 90; j < 180; j++)
{
    if( (theta_Be >= (0.008727 + j * 0.017453)) &&
        (theta_Be <= (0.008727 + (j + 1) * 0.017453)) )
    {
        for (int jz = 0; jz < 100; jz++)
        {
            if( ( ((Z_de > offset_backward + 5.0 + 0.0025
                    - (jz + 1) * 0.0500 ) ) && ( Z_de <=
                    (offset_backward + 5.0 - 0.0025 - jz * 0.0500) ))
                && ( Y_de >= offset_left) ) && (Y_de <=
                    (offset_left + 5.0)) )
            {
                ++angolosolido_de[4][jz][j];
            }
        }
    }
}
break;
} // closure of the ‘for’ on the event random generation

Also at backward angles the cases tel = 6, 7, 8 can be obtained from that
tel = 5 through opportune replacement of the parameters Y_de, Y_er and
offset_left.

Output data printing

for (int x1 = 0 ; x1 < 8 ; x1++)
{
    printf("%u\t", scattering_1[x1]);
```c
}
printf("\n\n");

for (int y2 = 0 ; y2 < 8 ; y2++)
{
  printf("%u\t", rearside_2[y2]);
}
printf("\n\n");
and similarly for the other output vectors.

for (int x1 = 0 ; x1 < 8 ; x1++)
{
  printf("Telescope number: %d\n", x1 + 1);
  for (int y1 = 0 ; y1 < 100; y1++)
  {
    printf("Strip No: %d\t", y1 + 1);
    int z1min, z1max;
    if(x1 < 4)
    {
      z1min = 0;
      z1max = 89;
    }
    else
    {
      z1min = 90;
      z1max = 179;
    }

    for (int z1 = z1min ; z1 <= z1max ; z1++)
    {
      printf("%f\t", (float) angolosolido_de[x1][y1][z1] / imax * 4 * PI *1000);
    }
    printf("\n");
  }
  printf("\n");
}

for (int x1 = 0 ; x1 < 8 ; x1++)
```
{  
printf("Telescope number: \%d\n", x1 + 1);
for (int y1 = 0 ; y1 < 100; y1++)
{
    printf("Strip No: \%d\t", y1 + 1);
    int z1min, z1max;
    if(x1 < 4)
    {
        z1min = 0;
        z1max = 89;
    }
    else
    {
        z1min = 90;
        z1max = 179;
    }

    for (int z1 = z1min ; z1 <= z1max ; z1++)
    {
        printf("\%f\t", (float) angolosolido_er[x1][y1][z1] / imax * 4 * PI *1000);
    }
    printf("\n");
}
printf("\n");
}
} // main closure
Appendix C

Data analysis with VISIM

All the data analysis has been performed with the package VISIM [Var89] using the C/C++ language and Linux as operating system. In this Appendix we present the subroutine coinc97.c used to the analysis of the experimental data collected at RIKEN to study the $^{11}$Be scattering process from a $^{209}$Bi target.

The second Section of this Appendix will illustrate for completeness the $^{11}$Be total energy spectra not shown in Sec. 4.6.1.

C.1 Analysis subroutine coinc97.c

C.1.1 Input/Output definition

Include files

```c
#include <stdio.h>
#include <unistd.h>
#include <stdlib.h>
#include "string.h"
#include "ev_Analysis.h"
#include "tree.h"
#include "1ndim.h"
```

Command line

The command line of the subroutine coinc97.c contains five arguments:

1. `argv[1]` input file (run65.evt, in the present case) containing the raw data;
2. **argv[2]** output file (run65.dat);

3. **argv[3]** run number (65);

4. **argv[4]** comment, between quotes, usually the name of used subroutine;

5. **argv[5]** configuration file (vconfig_riken_new) with the list of the used modules (AVI, ADC, TSI, TDC) and the correspondence between AVI board identification numbers, ADC module channels and EXODET $\Delta E$ and $E$ detectors.

```c
// coinc97 run65.evt run65.dat 65 "coinc97" vconfig_riken_new
/************************************************************************ /

Variables definition

```c
int main(int argc, char *argv[]) {

    int i, j, k, Type, Memory = 200, Inorm[32], Nparm, Numrun, IErr,
        NTypeMax, NType, IEndFlag = 0, FileNumber, ITe1, IAviDE, IAviER;
    char **Nompar, *cdummy = NULL, *Titman = NULL, *FileName = NULL;
    double x_de, y_de, z_de, x_er, y_er, z_er;
    double x_t = 0, y_t = 0, theta;
    int offset_ppac = 2000;
    int Strip_de, Strip_er, oker, okde1, okde2;
    double TDC[32];
    double Xa = 0, Xb = 0, Ya = 0, Yb = 0;
    double ETOFA, EIN, PLA, ToFA, ToFAc;

TDC and ADC calibration parameters

\[ TDC \text{ calibration (}^-0.3\text{ ns/ch)} \]

```c
double a_TDC[32] = {0.32383, 0.32010, 0.31516, 0.31461, 0.31786,
    0.32072, 0.30955, 0.32394, 0.30600, 0.32056,
    0.31585, 0.30979, 0.31133, 0.31842, 0.30883,
    0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0};

\[ \text{ADC calibration (slope }^-0.017\text{ MeV/ch, intercept 0-0.02 MeV)} \]

```c
float a_ADC[9][2] = { {0.0, 0.0 },
    {0.0164,0.0163}, {0.0172,0.0166},
```
float b_ADC[9][2] = {{0.0, 0.0},
{0.0167, 0.0165},
{0.0164, 0.0165},
{0.0168, 0.0162},
{0.0180, 0.0164},
{0.0180, 0.0164},
{0.0171, 0.0165}};

Input/Output file opening

Type = AN_VIPERS_FORMAT;
if(argc == 6)
{
  if(ev_AnOpenEvtFile(Type, (char *) argv[1]) < 0 ){
    printf("\n AN_NO_OPEN_FILE\n");
    exit(0);
  }
}
else
{
  printf("\n Usage: to be written....sorry\n");
  exit(1);
}

FileName = U_StringCopy((char *) argv[2]);
Numrun = atoi( (char *) argv[3]);
Titman = U_StringCopy((char *) argv[4]);

printf("\n Event File = {%s}", (char *) argv[1]);
printf("\n Run Number = %d", Numrun);
printf("\n Config File = {%s}", (char *) argv[5]);
printf("\n Output File Name = {%s}", FileName);
printf("\n Title = {%s}", Titman);

Output parameters definition

Nparm = 20;
Nompar = (char **) malloc(Nparm * sizeof(char *));
Nompar[0] = U_StringCopy("RUN"); // run number
Nompar[1] = U_StringCopy("TEL"); // telescope number
Nompar[2] = U_StringCopy("DE"); // DeltaE energy signal
Nompar[3] = U_StringCopy("ER"); // ER energy signal
Nompar[4] = U_StringCopy("NSDE"); // Delta E hit strip number
Nompar[5] = U_StringCopy("NSER"); // ER hit strip number
Nompar[6] = U_StringCopy("STDE"); // DeltaE hit strip
Nompar[7] = U_StringCopy("STER"); // ER hit strip
Nompar[8] = U_StringCopy("JTDE"); // Jitter Time DeltaE
Nompar[9] = U_StringCopy("TTDE"); // Time-over-Threshold DeltaE
Nompar[10] = U_StringCopy("JTER"); // Jitter Time ER
Nompar[12] = U_StringCopy("Theta"); // scattering angle
Nompar[13] = U_StringCopy("Beam"); // Beam ok signal
Nompar[14] = U_StringCopy("EIN"); // incident energy
Nompar[15] = U_StringCopy("OKER"); // ER good events
Nompar[16] = U_StringCopy("ETOT"); // total energy
Nompar[17] = U_StringCopy("XT"); // target coordinate (x)
Nompar[18] = U_StringCopy("YT"); // target coordinate (y)
Nompar[19] = U_StringCopy("OKDE"); // DE good events

printf("\n\n Parameters: ");
for(i = 0; i < Nparm; i++){
    printf("%s", Nompar[i]);
}

InitBucket(Memory, Nparm, &NTypeMax);
printf("\n\n NTMax = %d, NTypeMax = %d", NTMax, NTypeMax);
Nompar[1] = U_StringCopy("TEL");
FileNumber = -1;

C.1.2 RIPS signal readout

switch(Type){

    case AN_VIPERS_FORMAT:
        ev_ReadVipersConfig((char *) argv[5]);
        Inorm[0] = Numrun;

        while(ev_GetVipersEvent(gDAT) > 0){

            if(ev_MapVipersEvent(gDAT) != 0){
                printf("\n\n\n WRONG EVENT MAPPING\n");
            }
continue;
}

for(i = 1; i < Npar; i++){
    Inorm[i] = 0;
}

TDC calibration

    // TDC calibration

    for(i = 0; i < 32; i++){
        TDC[i] = a_TDC[i] * gTDC[i];
    }

Beam ok signal

As described in Sec. 4.3.4, a veto circuit between the signals coming from the F2 Plastic Scintillator and the F3 PPACs rejected pileup events between these two locations and ensured a correct event energy reconstruction. This module also provided a logical signal, named Beam ok signal, that was inserted in one TDC module channel.

    // Beam ok signal

    Inorm[13] = TDC[12];  // Beam ok

11Be incident energy reconstruction

This part of the code performs the 11Be incident energy reconstruction. The whole experimental procedure has been described in Sec. 4.3.

    // Beam energy reconstruction via Time-of-Flight

    PLA = (TDC[0] + TDC[1]) / 2;  // average of PLL and PLR
    ToFA = PLA - TDC[6];  // Time-of-Flight
    ToFac = -1.0717 * ToFA + 226.2124;  // calibrated Time-of-Flight

    if ((ToFac > 0) && (ToFac < 1024))
    {
        ETOFA = (0.5 * 10267 * pow(4.768 * 3.3357 / ToFac,2));
\[ EIN = -0.002726 \times \text{pow}(E\text{TOFA}, 2) + 1.432 \times E\text{TOFA} - 18.42; \]
\[
\text{if (EIN > 1)}
\begin{align*}
\text{Inorm[14]} &= \text{EIN} \times 10; \\
\end{align*}
\]

**PPAC calibration**

Position calibration of the two PPACs placed around the focus F3, following the procedure described in Sec. 4.4.1.

// PPAC coordinates

if( TDC[2] > 0 && TDC[7] > 0)
\[
Xa = 1.6 - 1.246 \times (TDC[2] - TDC[7]) / 2;
\]
if( TDC[8] > 0 && TDC[3] > 0)
\[
Ya = -1.0 - 1.254 \times (TDC[8] - TDC[3]) / 2;
\]
if( TDC[4] > 0 && TDC[9] > 0)
\[
Xb = -1.0 - 1.245 \times (TDC[4] - TDC[9]) / 2;
\]
if( TDC[10] > 0 && TDC[13] > 0)
\[
Yb = -0.7 - 1.239 \times (TDC[10] - TDC[13]) / 2;
\]

**Target position reconstruction (see Sec. 4.4.2)**

// target coordinates

\[
x_t = Xb + 682.5 / 304 + (Xb - Xa);
\]
\[
y_t = Yb + 682.5 / 304 + (Yb - Ya);
\]
\[
\text{Inorm[17]} = x_t + \text{offset_ppac};
\]
\[
\text{Inorm[18]} = y_t + \text{offset_ppac};
\]

// selection of the target central position

if( pow(y_t, 2) + pow(x_t, 2) < 400)

C.1.3 **EXODET event readout**

The readout electronics of the whole EXODET array has to consider both the energy signals coming from the $\Delta E$ and $E$ continuous backplanes and the position information collected by the detector segmented front sides and processed by the chip ASIC. We remind that for each $\Delta E$ ($E$) hit strip, the output data stream of the chip consists of (i) the number of the hit...
strip, named NSDE (NSER) in the present code, (ii) the Jitter Time, JTDE (JTER), and (iii) the Time-over-Threshold, TTDE (TTER). The macro first proceeds the readout of both the $\Delta E$ and $E$ energy signals, then analyzes the $\Delta E$ position information and finally those coming from the $E$ layer.

**$\Delta E$ and $E$ energy signal readout**

```c
for(i = 0; i < gN_TEL; i++){
    ITel = gTeles[i];
    if(gMapTel[ITel][0] == 4095) continue;
    Inorm[1] = ITel; // hit telescope
    Inorm[2] = (int)ceil((gMapTel[ITel][0] * a_ADC[ITel][0]
                        + b_ADC[ITel][0]) * 10); // DE energy signal
    Inorm[3] = (int)ceil((gMapTel[ITel][1] * a_ADC[ITel][1]
                        + b_ADC[ITel][1]) * 10); // ER energy signal
    Inorm[16] = Inorm[2] + Inorm[3]; // Event total energy
    IAviDE = gTelToAVI[0][ITel];
    IAviER = gTelToAVI[1][ITel];
    Inorm[4] = gNStrip[IAviDE]; // Number of strips in DE
    Inorm[5] = gNStrip[IAviER]; // Number of strips in ER

        { // beginning if on the DE and ER

**$\Delta E$ position readout**

During the analysis of the $\Delta E$ position information we defined an additional parameter, named OKDE. This parameter was increased when the event had the Jitter Time inside the correlation peak ($9 \leq JTDE \leq 11$) and Time-over-Threshold corresponding to the $^{11}$Be range ($7 \leq TTDE \leq 8$). According to the data analysis presented in Ch. 4, this parameters is a kind of $\Delta E$-“good event” label.

```c
if(gMapTel[ITel][1] == 4095)
    {
        okde1 = 0;
        Inorm[5] = 0; // Number of strips in ER
        for(j = 0; j < gNStrip[IAviDE]; j++){
            Inorm[6] = gStripHit[IAviDE][j];
            Strip_de = Inorm[6];
            Inorm[7] = 0;
            Strip_er = Inorm[7];
```
Inorm[8] = gStripMap[IAviDE][j][0]; // JTime
Inorm[9] = gStripMap[IAviDE][j][1]; // ToT

if( ((Inorm[8]>=9) && (Inorm[8]<=11))
    {
        okde1++;  
        Inorm[19] = okde1;
    }
else
    Inorm[19] = 0;

Inorm[10] = 0;
Inorm[11] = 0;
}
else
{
    okde2 = 0;
    for(j = 0; j < gNStrip[IAviDE]; j++){

        Inorm[6] = gStripHit[IAviDE][j];
        Strip_de = Inorm[6];
        Inorm[8] = gStripMap[IAviDE][j][0]; // JTime
        Inorm[9] = gStripMap[IAviDE][j][1]; // ToT

        if( ((Inorm[8]>=9) && (Inorm[8]<=11))
            {
                okde2++;  
                Inorm[19] = okde2;
            }
else
    Inorm[19] = 0;

oker = 0;

E position readout

Similarly to the readout of the \(\Delta E\) digital information, also in this case we defined a \(E\)-“good event” label, with the same gates as previously, \(i.e. \ 9 \leq \)
JTER \leq 11 and 7 \leq TTER \leq 8.

for(k = 0; k < gNStrip[IAviER]; k++){
    Inorm[7] = gStripHit[IAviER][k];
    Strip_er = Inorm[7];
    Inorm[10] = gStripMap[IAviER][k][0]; // JTime;
    Inorm[11] = gStripMap[IAviER][k][1]; // ToT;

        {
            oker++;
            Inorm[15] = oker;
        }
    else
        Inorm[15] = 0;

EXODET position reconstruction

The EXODET position hit by the scattered particle \((x_{de},y_{de},z_{de})\) is reconstructed from the hit target position \((x_{t},y_{t})\), the \(\Delta E\) hit strip number (Strip\_de) and the \(E\) hit strip number (Strip\_er). The position reconstruction obviously depends on the geometrical displacement of the hit telescope, as shown by the following code.

switch(ITel)
{
    case 1:
        x_de = 26.25;
        z_de = 6.45 + Strip_de * 0.5;
        x_er = 31.45;
        y_er = -24.0 + Strip_er * 0.5;
        z_er = (31.45 - x_t) / (26.25 - x_t) * z_de;
        // y_de calculation
        y_de = y_t + (26.25 - x_t) / (31.45 - x_t) * (y_er - y_t);
        break;
    case 2:
        y_de = 26.25;
        z_de = 6.45 + Strip_de * 0.5;
        x_er = 24.0 - Strip_er * 0.5;
        y_er = 31.45;
\[ z_{er} = (31.45 - y_t) / (26.25 - y_t) \times z_{de}; \]

// x_de calculation
\[
x_{de} = x_t + (26.25 - y_t) / (31.45 - y_t) \times (x_{er} - x_t);
\]

break;

// x_de calculation
\[
x_{de} = -26.25;
\]

z_{de} = 6.45 + Strip_{de} \times 0.5;

x_{er} = -31.45;

y_{er} = 24.0 - Strip_{er} \times 0.5;

z_{er} = (-31.45 - x_t) / (-26.25 - x_t) \times z_{de};

break;

case 3:

\[
\]

// y_de calculation
\[
y_{de} = y_t + (-26.25 - x_t) / (-31.45 - x_t) \times (y_{er} - y_t);
\]

break;

case 4:

\[
\]

// y_de calculation
\[
x_{de} = x_t + (-26.25 - y_t) / (-31.45 - y_t) \times (x_{er} - x_t);
\]

break;

case 5:

\[
\]

// y_de calculation
\[
y_{de} = y_t + (26.25 - x_t) / (31.45 - x_t) \times (y_{er} - y_t);
\]

break;

case 6:

\[
\]

// y_de calculation
\[
z_{er} = (31.45 - y_t) / (-26.25 - y_t) \times z_{de};
\]
// x_de calculation
x_de = x_t + (26.25 - y_t) / (31.45 - y_t) * (x_er - x_t);
break;
case 7:
x_de = -26.25;
z_de = -2.45 - Strip_de * 0.5;
x_er = -31.45;
y_er = -24.0 + Strip_er * 0.5;
z_er = (-31.45 - x_t) / (-26.25 - x_t) * z_de;
// y_de calculation
y_de = y_t + (-26.25 - x_t) / (-31.45 - x_t) * (y_er - y_t);
break;
case 8:
y_de = -26.25;
z_de = -2.45 - Strip_de * 0.5;
x_er = 24.0 - Strip_er * 0.5;
y_er = -31.45;
z_er = (-31.45 - y_t) / (-26.25 - y_t) * z_de;
// x_de calculation
x_de = x_t + (-26.25 - y_t) / (-31.45 - y_t) * (x_er - x_t);
break;
} // end switch(tel)

Scattering angle $\theta$ reconstruction

The reconstruction of the scattering angle $\theta$ is performed using the formula 4.6, that we rewrite according to the formalism employed in the code:

$$\theta = \arctan\left(\frac{\sqrt{(x_{de} - x_t)² + (y_{de} - y_t)²}}{z_{de}}\right)$$

(C.1)

The scattering angles are generally defined in the range $0^\circ$-$180^\circ$, whereas the natural codomain of the arctan function is between $-90^\circ$ and $90^\circ$. To overcome this problem we simply added $180^\circ$ to the resulting scattering angles smaller than $0^\circ$.

// theta angle reconstruction
theta = atan( sqrt( pow(x_de - x_t, 2) + pow(y_de - y_t, 2) ) / z_de ) * 180 / 3.1415;
if(theta >= 0)
    Inorm[12] = theta;
else
    Inorm[12] = 180 + theta;

Bucket filling

if( ( ((Inorm[8] >= 9) && (Inorm[8] <= 11)) // JTDE
{
    FillBucket(Inorm, &NType, &IEndFlag);
    if(IEndFlag == 1){
        printf("\n ***** CREATING NDIM FILE *****\n");
        printf("\n\n NType = %d NTypeMax = %d",
               NType, NTypeMax);
        FileNumber++;
        cdummy = (char *) malloc((strlen(FileName) + 10)
                * sizeof(char));
        sprintf(cdummy, "%s.%d", FileName, FileNumber);
        cdummy[strlen(cdummy)] = '\0';
        printf("\n%s\n", cdummy);
        printf("\n%d\n", Nparm);
        printf("\n%s\n", Titman);
        nd_CreateNdim(cdummy, Nparm, Nompar, Titman);
        free(cdummy);
        ZeroBucket();
        FillBucket(Inorm, &NType, &IEndFlag);
    }
} //end if on JTDE, TTDE, JTER and TTER parameters

} // end else

} // end if on the DE and ER parameters

} // end for

} // end if on the target central position
} // end while

if(IEndFlag == 0 && NType != 0){
    printf("\n ***** CREATING NDM FILE *****\n");
    printf("\n\nNType = %d NTypeMax = %d", NType, NTypeMax);
    FileNumber++;
    cdummy = (char *) malloc((strlen(FileName) + 10)
        * sizeof(char));
    if(FileNumber == 1)
        sprintf(cdummy, "%s", FileName);
    else
        sprintf(cdummy, "%s.%d", FileName, FileNumber);
    cdummy[strlen(cdummy)] = '\0';
    printf("\n%s\n", cdummy);
    printf("\n%d\n", Nparm);
    printf("\n%d\n", Titman);
    nd_CreateNdim(cdummy, Nparm, Nompar, Titman);
    free(cdummy);
}
break;

default:
    printf("\nNO FILE TYPE\n");
} // end switch(Type)

ev_AnCloseEvtFile();
ev_AnPrintEvtStatistics(stdout);
FileNumber++;
nd_NdimMergeFiles(FileName, FileNumber, &IErr);

return 0;
}

C.2 Total energy spectra

This Section will briefly present for completeness the $^{11}$Be total energy spectra for the system $^{11}$Be + $^{209}$Bi at 38-, 44-, 46- and 48-MeV beam energy, not shown in Sec. 4.6.1. These spectra have been obtained adding the energy
losses by the scattered $^{11}$Be ions in the $\Delta E$ and $E$ telescope stage.

38 MeV

![Graph showing energy spectrum](image)

**Figure C.1:** Total energy spectrum (sum of the energy lost in the $\Delta E$ and $E$ telescope stages) for the six angular bins used to evaluate the scattering differential cross section for the system $^{11}$Be + $^{209}$Bi at 38-MeV beam energy.
Figure C.2: Total energy spectrum (sum of the energy released in the ΔE and E telescope stages) for the six angular bins used to evaluate the scattering differential cross section for the system $^{11}\text{Be} + ^{209}\text{Bi}$ at 44-MeV beam energy.
46 MeV

![Energy Spectrum Graph](Figure C.3: Total energy spectrum (sum of the energy lost in the $\Delta E$ and $E$ telescope stages) for the six angular bins used to evaluate the the scattering differential cross section for the system $^{11}\text{Be} + ^{209}\text{Bi}$ at 46-MeV beam energy.)
C.2. Total energy spectra

48 MeV

![Graph showing total energy spectra for different angular bins.]

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*and more, much more than this,*  
*I did it my way.*  
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